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by

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ANTI-ROLLING TANK DESIGN

By

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ABSTRACT

The spectral analysis technique is used in order to predict the wave induced rolling motions of a ship in a seaway. Non-linear equations of motion for the ship, the anti-rolling tank, and for wave excitations are developed. Solution techniques for the equations are examined, and a linearized form is adopted. A design method using a digital computer program based upon numerical solution of the linearized equations is proposed and examined for a number of different representative ships. The design method is demonstrated to be capable for use to design passive anti-rolling tanks for ships and to determine feasibility of activated anti-rolling tanks.

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My thanks go to Dr. Dan Whitney who took the time to critically read and evaluate this paper. His contributions will be appreciated by anyone who tries to puzzle through the details of the text.

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Nomenclature

A_o	Area of fluid surface in one wing tank
A_1	Sectional area of cross duct
a	Area of any tank section perpendicular to l, $a = A_o$ at $l = 0, L$
B	Location of the center of buoyancy
B	Beam of ship
BG	Distance from the center of buoyancy to the center of gravity
B_t	Tank linear damping coefficient
b_t	Wing tank breadth
Cb	Block coefficient, $35\Delta/$ (L B T)
Cp	Longitudinal Prismatic coefficient, Cb/ Midships sectional area coefficient
d	Moment arm of section tank area a about G
D	$d(\underline{\qquad\quad})/dt$
D	$d(\underline{\qquad\quad})d\tau$
Di	Sway damping ordinates
e	Surface elevation of the sea, positive downwards
E	Tank movement
Eo(t)	Tank excitations
Fi	Sway damping frequency abscissas
G	Location of the center of gravity of the ship
g_i	Controlled feedback gains
GM	Metacentric height (transverse) of the ship, corrected for all free surface except for that of the tank.
h_t	Height of the wing tank
h_d	Height of the cross duct

$H^{1/3}$	Significant wave height, the average of the 1/3 highest waves
i	General subscript
i	Squareroot of -1
Ix	Moment of inertia of the ship about the x-axis
k	Wave constant, ω^2/g
K	Roll moment
K_p	First order roll velocity coefficient
K_ϕ	First order static righting coefficient
$K_{\dot{p}}$	First order roll acceleration coefficient
$K_{p\|p\|}$	Second order roll velocity coefficient
Ks	Dimensionless ship damping in roll
K_t	Dimensionless tank damping
K_q	Dimensionless ship quadratic damping in roll
K_3	Dimensionless cubic spring constant for ship in roll
K_y	Dimensionless ship damping in sway
KB	Distance from the keel to the center of buoyancy
KG	Distance from the keel to the center of gravity
l	Centerline distance along tank sections in the $+\eta$ direction
L	Endpoint of l
L	Length of ship
Le	Effective length of ship
Lo	The maximum fluid excursion along l from l = 0 in the η direction
l_t	Wing tank length
O	Origin of η coordinate

m	Mass of ship
p	Angular roll velocity, $\dot{\phi}$
$\dot{p}$	Angular roll acceleration, $\ddot{\phi}$
$p(t)$	Pump pressure
p^*	Dimensionless pump pressure, $p(t)/\rho gR$
r	Radial distance from G to a point along l
R	Distance from the centerline of the ship to the center of area A_o
s	Distance along wave from reference
S'	Weighted tank section area, $\int_o^L \left(\frac{Ao}{a}\right) dl$
S''	Weighted tank moment arm, $\int_o^L \left(\frac{d}{R}\right) dl$
S	Wave elevation spectral density function
t	Time
Tp	Pump lag time
t_b	Draft to the center of buoyancy
T	Draft of ship
U	Speed of ship in the x-direction
v	Sway velocity of ship
v^*	Dimensionless sway velocity, $v/\omega sB$
v_w	Horizontal wave velocity
Wt	Tank weight
x-y-z	Ship reference coordinate axes
x_o-y_o-z_o	Earth referenced coordinate axes
X	Heading angle parameter, $Le \lvert\cos(\beta)\rvert/\lambda$
y	Sway direction
Y	Sway force

Y_v	First order sway damping coefficient
$Y_{\dot{v}}$	First order sway inertia coefficient
z_h	Distance from G to B in the positive z direction
z_i	Sway coefficients in the expanded equations of motion
z_t	Distance from G to 0 in positive z direction
α	Parameter of the Pierson-Moskowitz Spectral Density function
αi	Dimensionless coupling parameters in the expanded equations of motion
β	Parameter of the Pierson-Moskowitz Spectral Density function
β	Heading angle, 180° for head seas
βi	Dimensionless coefficients of roll in the expanded equations of motion
δi	Dimensionless tank coefficients in the expanded equations of motion
Δ	Displacement of ship, long tons (2240 lb)
$\Delta(\_\_\_)_i$	Increment of the parameter
ε	Tank effectiveness parameter, $\phi^{1/3}_{max}$ (tank)/ $\phi^{1/3}_{max}$(no tank)
ϕ	Roll angle from the vertical
ϕ_R	Maximum static equilibrium angle
ψ	Waveslope angle
γi	Inertia constant
λb	Ship-sway coupling parameter
λo	V_w-sway coupling parameter
λy	$V_{\dot{w}}$-sway coupling parameter
λt	Ship-tank coupling parameter

μ	Dimensionless driving frequency, ω/ω_s
μ_t	Tank tuning parameter, ω_t / ω_s
μ_{st}	Ship-tank decoupling frequency, ω_{st}/ω_s
μ_{yt}	Sway decoupling frequency, $\sqrt{g/B}/\omega_s$
ω	Frequency
ω_e	Encounter frequency of the waves as seen by the moving ship
ω_o	Absolute frequency relative to the earth
ω_s	Ship natural roll frequency, $\sqrt{K\phi/(K_p - Ix)}$
ω_{st}	Ship-tank decoupling frequency
ω_t	Tank natural frequency
ω_y	$\sqrt{g/B}$
ρ	Mass density of tank fluid
ρ_{H_2O}	Mass density of seawater, 35 tons/ft^3
τ	Dimensionless time, $\omega_s t$

Note: Computer program symbols are given separately in Appendix C

I. Introduction

1. General Discussion

Successful roll stabilization of ships by means of fluid filled Anti-Rolling tanks requires a thorough analysis of the forces and moments involved. The author undertook this task with a goal of being able to use the results as a design tool of the naval architect. As such, the Anti-Rolling tank design method derived in this paper is intended to be a single step in the larger process of designing the whole ship.

Modeling the dynamics of a ship with an Anti-Rolling tank has been investigated by many authors of whom only a small percentage are represented in the reference list. Several problems have prevented their analyses from being complete or from possessing sufficient accuracy in the range of interest.

One central problem is that rolling is a resonance phenomenon. The ship is an underdamped dynamic system in roll, one which magnifies wave excitations to a certain degree. This may be proved by considering that it is well established for waves in the open ocean to have a maximum waveslope of about 16 degrees. Waves much steeper than 16 degrees are unstable and break up by forming whitecaps or similar turbulence. However, rolls of 20 or 30 degress or more are common even in moderate seas. Thus, for the normal range of ship parameters and dimensions, a resonant condition occurs within the frequency range of wave exciting forces.

To better understand the implications of being in a

resonant condition, consider the ship as a simple second order system in roll, with the coordinate ϕ representing roll angle.

$$(K\dot{p}-Ix)\ddot{\phi} + Kp\dot{\phi} + K\phi\phi = -\text{ (Wave excitations)}$$

The natural resonant frequency of the system is $\omega_s = \sqrt{\frac{K\phi}{(K\dot{p}-Ix)}}$ and it may be recognized that the amplitude of the roll motion at the resonant frequency is proportional to $\sqrt{K\phi(K\dot{p}-Ix)}/Kp$ Hence the magnitude of the rolling motion at resonance will be most strongly dependent upon the damping parameter Kp. Some analyses of ship motions have assumed a linear damping parameter for the ship, i.e. Kp = a constant. This is a good approximation for small motions or for specific conditions, but in the general case we expect Kp to be non-linear. Since roll damping involves viscous friction, we expect that there will be a Reynold's number dependency. Also, because rolling creates waves, there is added damping due to the radiation of wave disturbances from the ship. This radiation of wave energy has been shown to be frequency dependent in a complicated fashion.

Another problem is that rolling motion is not independent of other ship motions and therefore cannot be reasonably expressed as a simple second order system. For example, a ship at rest has been caused to assume an angle of roll which is different from its usual upright position. This roll angle causes the ship to have an unsymmetrical underwater shape, which induces a righting moment (assuming a stable ship). Obviously, any pitching motion would change the underwater shape

still further, unless the ship were prefectly spherical, and would induce a change in the value of the righting moment. It can be shown by similar arguments that roll is coupled to all of the other ship motions and displacements. However, extensive studies have shown that the degree of coupling is negligible in all instances, except for sway rate and sway acceleration, assuming a "normal" surface ship.

This paper will take an engineering approach towards analyzing rolling motion. The best available information on how to model non-linearities will be used in a workable system which allows calculations to be done in a reasonable amount of time. This approach was taken because of the iterative nature of the design process for Anti-Rolling tanks, which in turn is one step in a larger iterative design process for the entire ship. Therefore the analytical model of the Anti-Rolling tank had to be flexible enough to be "turned inside out" into a systhesis model by which a large range of parameters could be investigated by some search technique. This factor was the primary motivation behind taking an engineering approach to a sufficient design rather than a purely analytical approach towards a perfect dynamic model. It is hoped that this paper possesses a good enough balance between numerical estimation and reality to shed a little more light on a very complicated subject.

2. System of Units

The English system of units is used throughout this paper and in the computer program. The following English units

are assumed pertinent to all applicable quantities unless other units are indicated.

Time: seconds

Distance: feet

Mass: slugs

Angle: radians

Gravity constant: 32.2 ft. per $sec.^2$

Displacement: long tons (2240 lbs)

Force: pounds

Moment: foot-pounds

II. Derivation of the Equations of Motion

1. General Discussion

This chapter presents a detailed derivation of the equations of motion of the ship-tank system. The form of the equations is highly dependent upon the coordinate system chosen, which makes it important to understand section 2 and figures 1 and 2 before continuing with the text. For those who are interested only in the results, section 9 presents the final form of the equations and section 10 gives their non-dimensionalized equivalent. It is suggested that readers who are not interested in the derivations should still read section 2 before skipping to the results. This will ensure the reader has a reasonable feeling for the basic nomenclature of the equations before facing them.

The derivation of the equations of motion begins with Newton's second law of motion, action equals reaction. All of the terms which make up the reactive forces and moments are first listed. These terms are each broken down into their dependent variables. An expression for the reactions is then formed by taking the first partial derivative of the particular force or moment with respect to each of it's dependent variables. If the indicated partial derivatives were all constants the reaction terms would be exactly linear. Each of the partial derivatives is examined for this property, simplifications are made where possible and in several cases secondary parameters are introduced to further define particular coefficients. Enough documentation of relevant assumptions is made

to be able to change particular coefficient values whenever the state of the art permits. The excitation forces and moments are handled in the same fashion except that they have independent as well as dependent variables.

The last section of this chapter gives the summarized results as a set of simultaneous equations which are in non-dimensional form. This section is the key reference in understanding the rest of this paper. This is because the non-dimensional coefficient parameters found there constitute a kind of universal set which not only appears in most other literature on Anti-Rolling tanks (often with the same symbology) but also is used continuously throughout the rest of the paper.

2. Coordinate System

The coordinate axes for the ship are through the center of gravity of the ship and lie along the principal axes of inertia. They are x,y, and z. The reference coordinates with respect to the earth are x_o, y_o, and z_o; positive z_o direction is in the direction of gravity.

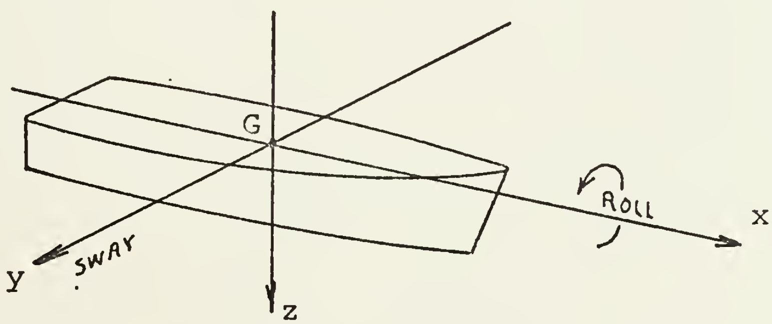


Figure 1. Ship-Oriented Coordinates

The tank coordinate, η, is relative to the ship and is positive in the direction shown on Figure 2. All of the symbols

shown on Figure 2 are in their positive sense.

The intent of this figure is to show the relative positions of points G, B, and O. G denotes the center of gravity of the ship with a filled Anti-Rolling tank (cross duct blocked) and serves as the reference for the ship oriented x-y-z axes. B, the center of buoyancy according to Archimede's principle, is assumed to be the point of application of the wave excitation forces and moments. O is the origin of the tank coordinate η. Also shown is the waveslope angle ψ and some of the basic Anti-Rolling tank geometry. Figure A-1 in appendix A gives a more complete picture of the Anti-Rolling tank by itself.

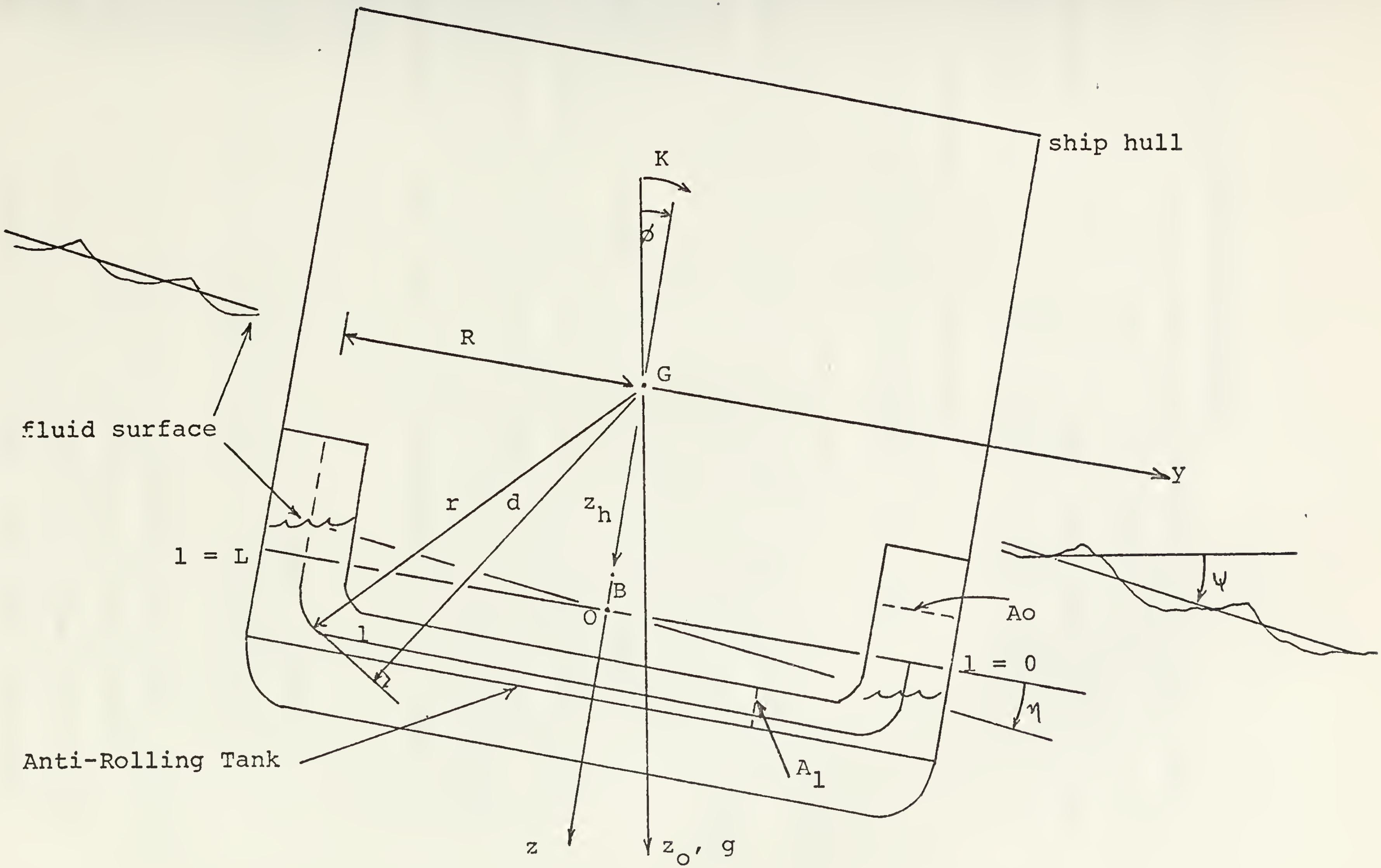


Figure 2. X-Z Plane, Looking Forward, Indicating Positive Direction of Axes, Forces and Moments

3. Reaction Forces and Moments.

For the ship-oriented coordinates a force and moment balance is done. K denotes a roll moment about the x-axis, Y denotes the force in the y direction, and E denotes the tank moment about an axis parallel to the x-axis.

$$\Sigma \text{ (Moments in } \phi \text{ direction)} = K = I\ddot{\phi}$$

$$\Sigma \text{ (Forces in the y direction)} = Y = m\dot{v}$$

For the tank coordinate, reference (4) uses a Lagrangian description of the potential, kinetic, and dissipation energies associated with the tank to arrive at its equation of motion. For the sake of brevity, only the results, which are well known, will be used in this paper.

Assuming a full tank with blocked ducts, there will be no interaction between the tank and roll or sway. In all other cases, there will be coupling between the three coordinates. This leads to an expression for K and Y as follows:

$$K = K(\text{tank}) + K(\text{hydrodynamic}) + K(\text{excitations})$$

$$Y = Y(\text{tank}) + Y(\text{hydrodynamic}) + Y(\text{excitations})$$

The hydrodynamic forces and moments are those imparted to the body of the ship by virtue of the ship motions occuring within and at the surface of a dense fluid (sea-water) in the presence of gravity. The tank forces and

moments are caused by the tank fluid shifting its weight transversely in the ship. Free surface waves at the surface of the ocean are the cause of excitation forces and moments.

This separation of terms in the force and moment equations may be carried further by noting which variables correspond to tank, hydrodynamic or excitation portions.

$$K \text{ or } Y(\text{tank}) = K \text{ or } Y\ (\eta,\ \dot{\eta},\ \ddot{\eta})$$

$$K \text{ or } Y(\text{hydrodynamic}) = K \text{ or } Y\ (\phi,\ \dot{\phi},\ \ddot{\phi},\ v,\ \dot{v})$$

$$K \text{ or } Y(\text{excitations}) = K \text{ or } Y\ (\psi)$$

The usual process of obtaining a first order model of a system is to take the first partial derivatives of each quantity (K or Y in this case) with respect to each of its arguments. After following the above procedure, combining terms where possible, and separating all the reactive forces and moments, including those of the tank, from the excitations, the roll and sway equations are obtained.

$$\text{(roll)}\quad (K\dot{v}D + Kv)v + [(K\dot{p} - Ix)D^2 + KpD + K\phi]\phi + K(\text{tank}) = -K(\text{excitations})$$

$$\text{(sway)}\quad [(Y\dot{v} - m)D + Yv]v + Y\dot{p}D^2 + YpD + Y\phi)\phi + Y(\text{tank}) = -Y(\text{excitations})$$

where: $D = d/dt$

$Kv = \partial/(\partial v)K$ $\qquad Yv = \partial/(\partial v)Y$

$K\dot{v} = \partial/(\partial \dot{v})K$ $\qquad$ etc.

$Kp = \partial/(\partial p)K$

$$K\dot{p} = \partial/(\partial\dot{p})K$$

$$K\phi = \partial/(\partial\phi)K$$

The first order equation of motion for the Anti-Rolling tank, as found in reference (4) is:

$$\text{(tank)} \quad \rho A_o R^2 [2D\dot{v} - (S''D^2 + 2g)\phi - (S'D^2 + B_T D/2\rho A_o R^2 + 2g)\eta] = -E_o(t)$$

S' and S" are geometry factors which are calculated in Appendix A.

The coupling terms between roll, sway, and the tank are:

$$K(\text{tank}) = K\eta\eta + K\dot{\eta}\dot{\eta} + K\ddot{\eta}\ddot{\eta}$$

$$Y(\text{tank}) = Y\eta\eta + Y\dot{\eta}\dot{\eta} + Y\ddot{\eta}\ddot{\eta}$$

Since the coupling terms must be symmetric in any energy conservative system, the tank equation gives us directly the indicated partial derivatives in the roll and sway equations.

$$K\ddot{\eta} = -\rho A_o R^2 S'' \qquad Y\ddot{\eta} = +2\rho A_o R^2$$

$$K\dot{\eta} = 0 \qquad Y\dot{\eta} = 0$$

$$K\eta = -2\rho g A_o R^2 \qquad Y\eta = 0$$

Using these results and taking the case of a purely passive Anti-Rolling tank ($E_o(t) = 0$) the linear equations of motion for the ship-tank system are formed. The result is

a fifth order differential equation which is linear if the assumption is made that the coefficients of all the terms are constant.

(roll) $(K\dot{v}D + Kv)v + [K\dot{p} - Ix)D^2 + KpD + K\phi]\phi - \rho A_o R^2 (S''D^2 + 2g)\eta = -K_o(t)$

(sway) $[(Y\dot{v} - m)D^2 + Yv]v + (Y\dot{p}D^2 + YpD)\phi + 2\rho A_o R^2 D^2 \eta = -Y_o(t)$

(tank) $2\rho A_o R^2 Dv - \rho A_o R^2 (S''D^2 + 2g)\phi - \rho A_o R^2 (S'D^2 + B_T/\rho A_o R^2 D + 2g)\eta = -E_o(t) = 0$

In order to add a control system to the tank (mathematically) it is necessary to set the right hand side of the tank equation, $-E_0(t)$, equal to $-A_o Rp(t)$; $p(t)$ is the pump pressure developed across a control effector in the tank duct, with a $(+)p$ direction in the $(-)\eta$ direction. A separate equation for the dynamics of the pump must be included in the equations of motion. A simplified form of $p(t)$ which should be sufficient in the general case is to consider that any control effector can be characterized in a gross sense by an equivalent first-order time lag. This leads to the equation:

$$T_p \dot{p}(t) + p(t) = p_c(t)$$

$$p_c(t) = f(\phi, \dot{\phi}, v, \eta, \dot{\eta}, \psi)$$

$$p_c(t) = \begin{cases} g_1\phi + g_2\dot{\phi} + g_3\eta + g_4\dot{\eta} + g_5 v + g_6\psi, & |p_c| \le p_{max} \\ \pm p_{max}, & |g_1\phi + \ldots + g_6\psi| > p_{max} \end{cases}$$

The expression for $p_c(t)$ was chosen based on linear regulator theory which allows feedback of all the state variables, plus the excitation ψ. In practice, ψ is random and unmeasureable (economically), and of the state variables, the roll and tank variables are the easiest to measure and the most strongly coupled. This reduces the controller equation down to:

$$p_c(t) = g_1\phi + g_2\dot{\phi} + g_3\eta + g_4\dot{\eta}, \ |p_c| < p_{max}$$
$$p_c(t) = \pm p_{max}, \ |g_1\phi + g_2\dot{\phi} + g_3\eta + g_4\dot{\eta}| \geq p_{max}$$

A useful equation which must be considered along with the above set of state equations is an estimation of the power requirements of the control effector. Using the relationship that power is the product of pressure differential and volume flow, and taking into account pump efficiency, power requirements are calculated by:

$$\text{Pump Power} = (1/e_p)p(t)A_oR\dot{\eta}$$
$$\text{where:} \quad e_p \simeq 0.5 - 0.8$$

4. Simplifications of the Hydrodynamic Coefficients.

Assume that the roll and sway motions of the ship as seen at the center of buoyance are uncoupled; that is, a roll motion about the center of buoyancy induces no swaying and likewise a sway motion at the center of buoyancy induces no rolling there. It may be easily imagined that

this is nearly true. Using this basic assumption, many simplifications in the terms of the hydrodynamic coefficients are possible. For example, a roll motion about the x-axis, which passes through the center of gravity, will be seen at the center of buoyancy as a roll motion plus a sway motion. The hydrodynamic resistance to sway at the center of buoyancy is felt as a sway force, but does not couple back into roll, with this assumption. Using the (') notation to indicate that a particular coefficient is taken at the center of buoyancy instead of being referenced to the center of gravity (which is the center of the roll and sway axes) the values of the following hydrodynamic coefficients are determined.

$$Y'v = Yv \qquad Yp = Y'p = -z_h Yv$$

$$Y'\dot{v} = Y\dot{v} \qquad Y\dot{p} = Y'\dot{p} = -z_h Y\dot{v}$$

$$K'\phi = K\phi \qquad Kv = K'v = -z_h Yv$$

$$K\dot{v} = K'\dot{v} = -z_h Y\dot{v}$$

Introduction of the above relationships shows which hydrodynamic coefficients must be known as a minimum in order to solve the equations of motion of the ship-tank system. Notice that these coefficients are all defined as being determined with respect to the x-y-z coordinate system about the center of gravity of the ship.

$K\phi$ = Static righting moment characteristic of the ship.

Kp = Linear damping in roll.

$K\dot{p}$ = Added inertia of the ship in roll to accomodate water flow.

Yv = Linear damping in sway.

$Y\dot{v}$ = Added inertia of the ship in sway to accomodate water flow.

5. Tank Coefficients.

The parameters which determine the coefficients of the tank variables in the linear equations of motion are:

$\rho A_O R^2$ = Characteristic of tank size.

S'' = Function of tank geometry = $\int_0^L (d/R)dl$

S' = Function of tank geometry = $\int_0^L (A_0/a)dl$

B_t = Tank linear damping.

S' and S'' are the weighted moment arm and sectional area, respectively, of the tank fluid. They are familiar terms in literature on hydraulic engineering. $\rho A_O R^2$ has the units of mass times a moment arm and is a characteristic tank size or inertia term.

The above tank parameters are all free to be chosen. However, it can be noticed that S'' and S' fall within the following range of possible values.

$$1 \leq S' \leq \infty$$

$$-1 \leq S'' \leq 1$$

$$|B_t| \geq 0.05(\sqrt{2gS'}\ \rho A_0 R^2)$$

For the case of S' getting very large, the tank dynamics slow down due to the increased fluid inertia. The extreme case is the equivalent of the tank being just an additional weight aboard ship. The sign change of S" is accomplished by changing the center of gravity of the tank relative vertically to the center of gravity of the ship. A negative sign indicates a high tank, a zero value occurs near the center of gravity of the ship, and a positive sign is caused by a low tank.

The damping parameter of the tank, B_t, is free to be chosen as long as it exceeds the approximate minimum shown above, say about 5 percent of critical damping. Later analyses will show that a good passive tank will incorporate far more damping than this. Otherwise, undesirable secondary resonance peaks appear in the frequency response of the ship-tank system. Therefore, for practical purposes B_t may be considered a free parameter with the stipulation that in the final design of the tank suitable damping devices such as sharp corners, baffles, etc. are used to achieve the desired degree of energy dissipation. B_t may be calculated or predicted accurately enough with standard hydraulic engineering 'pipe friction' tables.

6. <u>Effector Pump Parameters</u>.

The maximum efficient effector size can be easily

estimated for any particular tank geometry, since the effector pump is installed in the cross ducting. This will enable the choice of a particular sized pump which we have chosen to characterize by its maximum pressure, p_{max}, and lag time, T_p. There is no particular requirement for p_{max}, except that p_{max} = (Power, max.)$e_p/A_o R\dot{\eta}$ max. For T_p however there is the requirement that the pump time lag must be less than the natural resonant frequency of the tank for the pump to have much useful effect. A pump with more lag will introduce instabilities into the control system.

7. Non-Linearities.

The hydrodynamic coefficients obviously all depend upon ship speed because they are caused by changing the flow field of the water about the ship. Of these, the damping parameters, Yv and Kp, are the most speed dependent because they involve the dynamic lift in the horizontal plane generated by the ship hull serving as an inefficient lifting body. An angle of attack is imparted to the mean water flow by the relative motion between the ship and the water in the horizontal plane determined by the vector addition of sway rate and forward speed. The resultant lift force is felt as a resisting force in the sway direction. The static righting moment coefficient, $K\phi$, is the least speed dependent because the only change from zero speed conditions is that the constant water pressure surfaces, such as the air-water inter-

face, are no longer planar, but are slightly folded into a wave pattern. Based on these arguments, and with no further proof, it was decided to adopt Kp and Yv as ship speed dependent, and the other hydrodynamic coefficients as speed independent.

Because the magnitude of the roll angle at resonant frequenty is much larger than its average value, the value of K_ϕ does not remain reasonably constant, say within 5-10%, within its expected range. Since the well known curve of static K versus ϕ is anti-symmetric, it was decided that it would be sufficient to adopt an odd polynomial expression for K_ϕ, as was done in reference (7). The $K_\phi \phi$ term is replaced by:

$$K_\phi \phi + K_{\phi 3} \phi^3$$

The $K_{\phi 3}$ coefficient may be simply evaluated by noting that the value of K is zero when ϕ is at the maximum static equilibrium angle, ϕ_R . This fixes the value of $K_{\phi 3}$ as $-K_\phi/\phi_R{}^2$. This can be even further simplified by approximately ϕ_R by about 80° or 1.4 radians. This enables the use of $K_{\phi 3} = -K_\phi/2$ if better information is lacking.

Part of the energy dissipation caused by roll motion is due to viscous friction resisting the relative motion of the skin of the ship to the water. Since viscous friction is Reynolds Number dependent, it must be roll velocity dependent. This component of roll damping can be calculated by standard hydraulic engineering 'friction factor' tables to include skin friction and the eddy-making resistance of the bilge keels. However, this usage precludes us from making the odd polynomial

approximation because the standard tables are all of the form $1/2\rho$(area)(velocity^2)(friction factor). This forces the adoption of replacing $K_p p$ by:

$$K_p p + K_{p|p|} p|p|$$

The absolute value sign is necessary to make the symmetric p^2 function an anti-symmetric function. Since the $K_{p|p|} p|p|$ term is primarily due to transverse flow meeting the tremendous flow obstruction of the bilge keels, and not for most cases much influenced by forward speed, it was decided to let this quantity remain independent of ship speed.

Another form of energy dissipation by rolling or swaying motion is found in the formation of surface waves which radiate outward from the ship. This energy dissipation has been estimated, using potential flow theory, for simple ellipsoidal bodies which resemble the underwater form of ships, and has been found to depend on ship speed and the frequency of motion (assuming harmonic motion). As would be expected, sway generates larger waves for an average hull form and therefore this effect is more pronounced for sway. Thus, it was decided to approximate sway damping as:

$$(Y_{v1}(\omega,U) + Y_{v2}(U))v;$$

where Y_{v_1} is the component associated with wave formation and Y_{v_2} is the component associated with the hull acting as a lifting surface in the horizontal plane. Roll damping may be well enough approximated by:

$$K_p p + K_{p|p|} p|p|;$$

where K_p is calculable by strip theory, or may be estimated from Appendix B as about .05 times critical damping. $K_{p|p|}$ can be estimated from friction factor tables at an average Reynolds number. If the above seems inconsistent, it should be recalled that roll damping is much less than the amount of sway damping that is coupled into roll by the equations of motion. Thus, the extra care is taken in the estimation of sway damping vice roll damping.

To determine the above coefficients exactly requires that extensive model testing be conducted. Since it is envisioned that the method proposed by this paper is done at a stage of design prior to any such detailed knowledge of the ship, the coefficients must be arrived at either by empirical data derived from previous similar designs, or else by estimation from their theoretical values for similar geometric forms. An appendix to this paper gives detailed examples of how to make these estimations.

As far as the tank non-linearities are concerned, the restriction on the maximum tank angle, η, by the boundaries of the tank is the most important of the non-linearities. The coupling parameters S' and S" remain almost constant for any tank geometry as long as centerline symmetry is preserved. However, a saturation effect is observed whenever the fluid in the tank reaches the tank top because at that point no more fluid transfer

athwartships is allowed so that the tank moment reaches a maximum. To account for saturation in the equations of motion there are two methods. An equivalent linear 'spring constant' may be used; to do this change S' such that the same energy storage for a given tank angle is achieved for a fictitious unsaturated tank as is realized in the real tank. Another method is to change the equations of motion at the point that the tank angle saturates. After saturation, consider that $\dot{\eta} = \ddot{\eta} = 0$. The normal tank dynamics resume again when the fluid can run 'downhill'. This condition is met in the static case for $\eta + \phi > 0$. In the dynamic case, the dynamic head must be included along with the pump pressure, if present. The condition for resumption of normal tank dynamics then is:

$$2\rho A_O R^2 Dv - \rho A_O R^2 (S''D^2 + 2g)\phi - 2\rho A_O R^2 g\eta + A_O R p(t) > 0.$$

The tank damping parameter, B_t, of course has non-linearities associated with it due to the nature of the damping devices used in the tank. However, as mentioned earlier, because we are interested in tank design, B_t can be considered a linear coefficient independent of the other variables with the stipulation that the amount of linear damping must be later designed into the tank by some means. It is reasonable to expect that an amount of damping in the same order of magnitude as critical damping is easily achievable.

8. Excitations

In order to keep the exciting function one-dimensional, it was arbitrarily decided to reference all the excitations to the waveslope. This was mostly done to be able to compare results with previous work. The waveslope used was not the surface waveslope, but an average value which the ship senses (remembering that surface waves are a local phenomenon which decay exponentially with depth). This average value is often taken at the half-draft of the ship, but to be consistent with hydrostatics as well as hydrodynamics, I chose to use an average value as the value at the center of hydrostatic buoyancy, which is nearly the half-draft. Also, this places the excitations at the center of buoyancy, and in accordance with a previous assumption about the non-coupling of sway and roll at this location, this neatly separates roll and sway excitations.

It is now necessary to review linear wave theory for simple harmonic ocean waves in deep water. A later assumption to be made is that the real ocean is built from a spectrum of these simple harmonic waves.

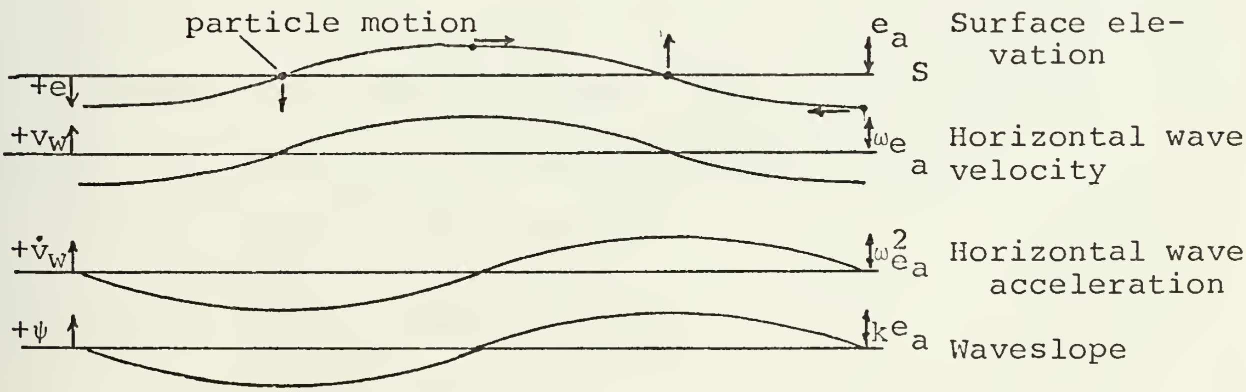


Figure 3. Wave Profile

Figure 3 is arrived at from the equation of surface elevation of an harmonic water wave, given in reference 3, which corresponds to a surface of constant (atmospheric) pressure. Horizontal wave velocity, acceleration, and waveslope are found by taking the appropriate partial derivatives.

$$e = e_a \cos(ks-\omega t)$$

$$v_w = \partial e/\partial t + 90° ! = -\omega e_a \cos(ks-\omega t)$$

$$\dot{v}_w = \partial v_w/\partial t = -\omega^2 e_a \sin(ks-\omega t)$$

$$\psi = \partial e/\partial s = -ke_a \sin(ks-\omega t)$$

An important factor to notice from the above math is that at any given point in the wave ψ and $\dot{v}_w$ are in phase with each other but are out of phase with the wave horizontal velocity v_w .

For simplicity, the wave variables may be expressed in complex form, with the implication that the quantity indicated is the real part of the complex expression.

$$e = e_a \exp(i(ks-\omega t))$$

$$v_w = -\omega e; \quad \dot{v}_w = i\omega^2 e; \ \psi = ike$$

$$v_w/\psi = -\omega/ik = ig/\omega$$

$$\dot{v}_w/\psi = \omega^2/k = g$$

The last two relationships fix v_w and $\dot{v}_w$ as functions of ψ.

Assuming that the excitations occur at the center of buoyancy, and using the same prescription as before:

$$K_o(t) = K'_\psi \psi + K'_{v_w} v_w + K'_{\dot{v}_w} \dot{v}_w$$

$$Y_o(t) = Y'_\psi \psi + Y'_{v_w} v_w + Y'_{\dot{v}_w} \dot{v}_w$$

There is an unrealistic assumption in the above derivation, however, in that the horizontal wave motions are not impinging on a static ship. They are actually influencing a moving ship, which suggests that for the equation to be valid, v_W and $\dot{v}_W$ must be replaced by $v_W - v'$ and $\dot{v}_W - \dot{v}'$, respectively.

By making the assumption that the hydrodynamic coefficients of the water moving past the ship are the same as for the ship moving past the water, and by adopting the classical assumption of Froude that $K'_\psi = K_\phi$, all of the coefficients are determined. Notice that Y'_ψ vanishes due to a previous assumption, but that K'_{v_W} and $K'_{\dot{v}_W}$ cannot vanish unless the moment arm, z_h, between the center of buoyancy and the center of gravity (the reference for the x-y-z axes) vanishes. With all of the above assumptions and conditions included, the excitations may now be found.

$$v' = v - z_h p$$

$$\dot{v}' = \dot{v} - z_h \dot{p}$$

$$K_O(t) = K_\phi \psi + zhYv(vw - v + zhp) + zhY\dot{v}(\dot{v}w - \dot{v} + zh\dot{p})$$

$$Y_O(t) = Yv(vw - v + zhp) - Y\dot{v}\ (\dot{v}w - \dot{v} + zh\dot{p})$$

All of the terms in the excitations, except for those involving $\psi, v_W(\psi)$ and $\dot{v}_W(\psi)$, may be moved to the left-hand side of the equations of motion when introducing this expression for the excitations into the equations of motion. When combined with like terms, this doubles the value of some of the hydrodynamic coefficients, and introduces coupling terms into some others. Also, the excitation expression added two new parameters instead of the one desired--the waveslope, and the phasing

difference between waveslope and horizontal wave motions.

9. Equations of Motion

The results of the previous sections are summarized to completely form the equations of motion for an anti-rolling tank.

(roll) $[(K\dot{p}-Ix)D^2 + KpD + K_{\phi}]\phi - zh[Y\dot{v}D + Yv_1(\omega,U) + Yv_2(U)]v$

$- \rho A_oR^2(S''D^2 + 2g)\eta + K_{p|p|}p|p| - K_{\phi/2}\phi^3 = -K_o(t)$

(sway) $zh[Y\dot{v}D^2 + (Yv_1(\omega,U) + Yv_2(U))D]\phi + [(Y\dot{v}-m)D$

$+ Yv_1(w,U) + Yv_2(U)]v + 2\rho A_oR^2D^2\eta = -Y_o(t)$

(tank) $-\rho A_oR^2(S''D^2 + 2g)_{\phi} + 2\rho A_oR^2Dv$

$-\rho A_oR^2(S'D^2 + (Bt/\rho A_oR^2)D + 2g)\eta = 0;$ or$- A_oRp(t)$

(effector) $(TpD + 1)p(t) = (g_1 + g_2D)\phi + (g_3 + g_4D)v,$

$|\Sigma g_ix_i| \leq P_{max} \pm P_{max}$, otherwise

(excitations) $K_o(t) = -K\phi\psi + zh(Yv_1 + Yv_2)(v_w - v + zhp)$

$+ zhY\dot{v}(\dot{v}_w - \dot{v} + zh\dot{p})$

$Y_o(t) = -(Yv_1 + Yv_2)(v_w - v + zhp) - Y\dot{v}(\dot{v}_w - \dot{v} + zh\dot{p})$

where: $\dot{\eta} = \ddot{\eta} = 0$ and $|\eta| = \eta max$, whenever $|\eta|$ exceeds ηmax.; the condition for returning to $\dot{\eta}$ and $\ddot{\eta}$ free is whenever:

$$2\rho AoR^2Dv - \rho AoR^2[S''D^2 + 2g]\phi - 2\rho gAoR^2\eta + AoRp(t) > 0;$$

with the initial condition $\eta = \pm \eta_{max}$ and $\dot{\eta} = 0$.

These equations may be linearized by dropping the terms involving ϕ^3 and $p|p|$, and the restriction on tank angle and pump pressure. A more convenient way to express these equations is to non-dimensionalize them; the results in non-dimensional form represent a family of solutions which help to give an insight of the general design problem.

10. Non-Dimensional Equations of Motion

Because further analysis in this paper uses mostly the linearized version of the equations, only the linear terms are given below. A paragraph at the end of this section gives the non-linear terms as well.

Dimensionless variables were chosen as follows:

$\tau = \omega_s t$

$v^* = v/\omega_s B$

$p^* = p(t) 2\rho g R$

ϕ , η , ψ in radians

$D = d(\ \ )/d\tau$

The linearized equations, with complex notation for the excitations, are:

(roll) $$[(1 + \lambda_b{}^2\gamma_1)D^2 + (K_s + \lambda_b{}^2K_y)D + 1]\phi - 2\lambda_b[\gamma_1 D + K_y]v^* + \lambda_t(D^2/\mu_{st}{}^2 + 1)\eta = [(1 - \lambda_b\lambda_y) - i\lambda_b\lambda_o]\ \psi(\tau)$$

(sway) $$-2\lambda b[\gamma_1 D^2 + K_y D]\phi + (\gamma_2 D + 2K_y)\ v^* - \lambda t D^2/\ \mu y_t{}^2\eta = (\lambda y + i\lambda o)\ \psi(\tau)$$

(tank) $$\lambda t(D^2/\mu_{st}2 + 1)\phi - \lambda t D/\mu_{yt}2\ v^* + \lambda t(D^2/\mu_t 2 + K_t D/\mu t + 1)\eta = 0;\ \text{or}\ \lambda t\ p^*(\tau)$$

(effector) $$2\rho g R(\omega_s T p + 1)p^* = (g_1 + \omega_s g_2 D)\phi + (g_3 + \omega_s g_4 D)\eta$$

Coupling constants

$\lambda t = -2\rho g A o R^2/K_\phi$

$\lambda b = zh/B$

$\lambda y = Y_{\dot{v}} gB/K_\phi$

$\lambda o = (Y_{v1} + Y_{v2})gB/\omega o K_\phi$

Damping constants

$K_s = K_p/\sqrt{K_\phi(K\dot{p} - I_x)}$

$K_y = B^2 Y_v \sqrt{K_\phi(K\dot{p} - I_x)}$

$K_t = B_t/\rho A o R^2 \sqrt{2g\ S'}$

Inertia constants

$\lambda_1 = B^2 Y\dot{v}/(K\dot{p} - I_x)$

$\lambda_2 = B^2(2Y_{\dot{v}} - m)/(K\dot{p} - I_x)$

Frequency constants

$\mu_t = \omega t/\omega_s$

$\mu_{st} = \omega st/\omega s$

$\mu yt = \sqrt{g/B}/\omega_s$

Spring constant (ship)

$K_\phi = -mgGM$

In order to preserve the non-linear equations, the following terms need be added to the indicated equation, and also the restriction on tank angle and pump pressure applies.

(roll) $K_q p|p| + K3\phi^3$

where: $K_q = (\omega s^2\ K_{p|p|})/K_\phi$

$K^3 = 1/2$, or $1/\phi_R{}^2$

There is one great idiosyncracy of these equations which would pass unnoticed were it not for the presence of the ω_o term in the wave velocity - sway coupling constant, λ_o . Here, ω_o refers to the absolute frequency of the waves relative to the earth. The rest of the dynamics will occur at a steady-rate frequency which is the encounter frequency that the ship senses in the waves. This encounter frequency is a function of both ship speed and relative heading angle, and is further examined in the appendix on excitation spectra. The peculiarity is that the ship is sensitive to both relative and absolute excitation frequencies, which is a direct result of the choice of sway excitations.

III. Excitation Spectrum

1. General Discussion

The preceeding chapter completely defined the anti-rolling tank problem with the exception of the single independent variable ψ (time). This chapter is dedicated towards obtaining a realistic expression for ψ. Ocean waves are essentially a random process. To account for the statistics of the wave process it was necessary to adopt an accepted empirical spectral (implying frequency) distribution function which had the disadvantage of being an elevation distribution instead of a slope distribution. The main part of the text of the chapter deals with making corrections and additions to this distribution function.

It is first changed into a waveslope distribution by use of linear gravity wave theory. Correction factors are derived for the amount of submergence of the ship's hull, for the relative heading of the ship with respect to wave progression, for the wave encounter frequency shift--akin to the familiar doppler shift--and for the shape of the ship's hull. Because of the highly speculative nature of some of the assumptions, particular attention is paid to their expected range of validity. All of the factors are then combined to form a complicated expression for the excitations which can be inserted into the equations of motion to complete the model of the ship-tank system in ocean waves.

This strategy was adopted because it lumped as many of the parameters which are independent of the anti-rolling tank design as possible into a single expression for excitations. This could then be used as an input forcing function. The resultant form of $\psi\cdot$(time) had the peculiar but advantageous properties of being "steady state", i.e. a combination of regular harmonic waves, and linear while only requiring that the various auxiliary component parameters be analytic. Thus the final derived expression for the excitation spectrum ψ included most of the design independent parameters as well as the statistical distribution of waveslope.

2. <u>Waveslope Distribution</u>

In order to describe the wave excitations of the sea surface it was decided to use the empirically derived Pierson-Moskowitz wave amplitutde spectrum. This is a standard analytical formulation which seems to adequately describe the real life wave amplitude spectrum for fully developed seas. The statistical distribution of wave amplitude over the band of wave frequency (0 to ∞) is assumed to depend on just one parameter, significant waveheight, $H^{1/3}$. Significant waveheight is commonly used as a measure of sea state. To be more specific, $e^2/\delta\omega$, wave amplitude squared per unit frequency bandwidth, is a function of frequency only for any given sea state. For a more complete description of this

spectral density function, its implications and proven uses, consult references 3 and 11.

$$e^2/\delta\omega = 0.0081\ g^2\omega^{-5}\ \exp(-\beta/\omega^4)$$

$$\beta = 0.0324\ g2/(H^{1/3})^2$$

$$g = 32.2\ \text{ft/sec.}^2$$

Using linear wave theory, it was previously shown that:

$$|\psi| = |\partial e/\partial s| = ke$$

where: $k = \omega^2/g$

Making the appropriate substitutions, the spectral density function for waveslope is found.

$$\psi^2/\delta\omega = k^2 S_{\xi\xi}(\omega) = \alpha g^{-2}\omega^{-1} e^{(-\beta/\omega^4)}$$

The mean square value of the waveslope spectrum is found by integrating the spectral density function over the entire frequency range.

$$\overline{\psi}^2 = \int_0^\infty k^2 S_{\xi\xi}(\omega)\,d\omega$$

The root mean square value (RMS) of wave slope is found by $\psi_{rms} = \sqrt{\overline{\psi^2}}$. By the statistics of a random process with an assumed Rayleigh distribution, the average of the 1/N highest values of ψ are given in reference 1:

$$\psi^{1/N} = f(1/N) \cdot \psi_{rms}$$

1/N	f(1/N)
1/1	1.25
1/3	2.00
1/10	2.55
1/100	3.34
$1/10^6$	5.27

It should be pointed out that the above numbers refer to the waveslope angle measured from the horizontal, and not to the "crest to trough" condition usually associated with waveheights.

Following the convention that "significant" means the average of the 1/3 highest values of the process, the significant waveheight is:

$$\psi^{1/3} = 2.0\psi_{rms}$$

This represents an acceptable peak value, for engineering purposes, of the random waveslope process. Notice that

the mean square value has a different relationship in this random process than for the deterministic case in which peak value is $\sqrt{2}$ times the RMS value.

3. Depth Correction

The surface elevation of the sea is a constant pressure surface. This pressure decays exponentially with depth until at a certain depth, the surface waves are not felt. By linear theory, this exponential decay factor is $e^{-\omega^2 zo/g}$, where z_o is the depth from mean surface elevation.

Since the center of buoyancy of the ship is the point through which wave excitations are induced in the static case, and for small amplitude excitations (i.e. nearly static case), a reasonable approach towards excitations is to apply them at the center of buoyancy. The only alternative to this is to carry out the integration of the pressure distribution over the entire immersed area of the hull. By the above reasoning, the spectral density function which the ship feels may be approximated by:

$$e^{-2tb\omega^2/g} \, S_{\xi\xi}(\omega);$$

where t_b is the draft to the center of buoyancy.

This correction factor for depth is shown in figure 4. Notice that for super ships, with drafts of 80+ ft and t_b of about 40 ft, waves are practically not felt in roll

excitation. For larger ships, say 20,000+ tons, all wave lengths less than about 50-100 ft. are unfelt, and are non-interactive with the ship.

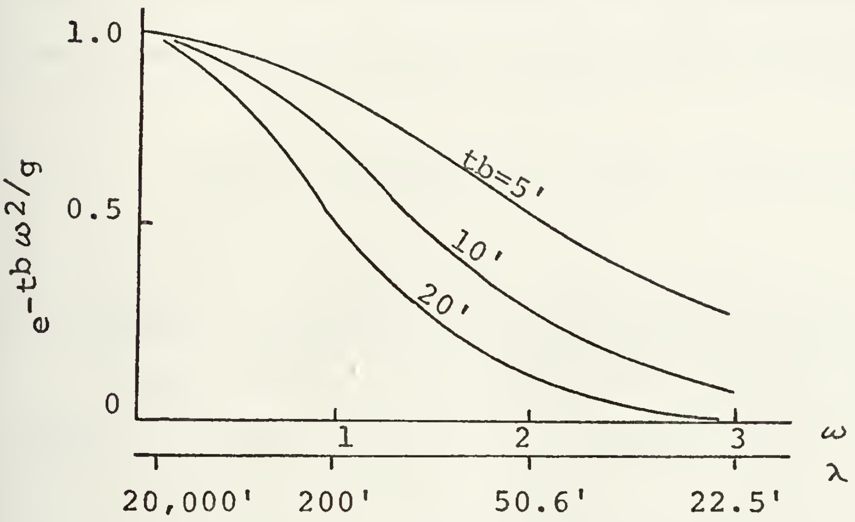


Figure 4. Depth Correction

4. Encounter Frequency

Since the ship forward speed and wave advance are both vector quantities, relative encounter frequency of the ship with the waves changes with orientation. Denote heading angle of the ship into the waves by β (β is 180° in head seas). The applicable relationship is:

$$\omega_e = \omega_o(1 - \omega_o U \cos \beta/g)$$

5. Transverse Waveslope

The equations of motion were developed for roll and sway coordinates which have entirely a transverse sense with respect to the ship. It can be shown that for waves progressing at some other direction than transversely to the ship the effective waveslope in the transverse direction is:

$$\psi(\beta) = \sin(\beta); \quad \beta \text{ is } 90° \text{ for beam seas.}$$

6. Region of Validity of Excitation Spectrum

Combining all the corrections to the original waveslope spectral density function, an expanded function is formed.

$$\psi^2/\delta\omega(\omega,\ t_b,\ \beta) \ = \ \sin^2(\beta)\ e^{-2t_b\omega^2/g}\ S_{\xi\xi}(\omega)$$

The expanded form of spectral density is plotted on Figure 6 for beam seas ($\sin(\beta) = 1$) for the normal range of expected sea states. A glance at the figures shows that for all ships with a draft of 5 feet or greater, the desired range of integration of $S_{\xi\xi}$ is $0.2 \leq \omega \leq 3.0$ to establish ψ_{rms}. The limits on ω impose the corresponding wavelength to fall within $20 \leq \lambda \leq 20{,}000$ feet.

To determine the range of β possible, a visualization of the ship in long crested waves is helpful.

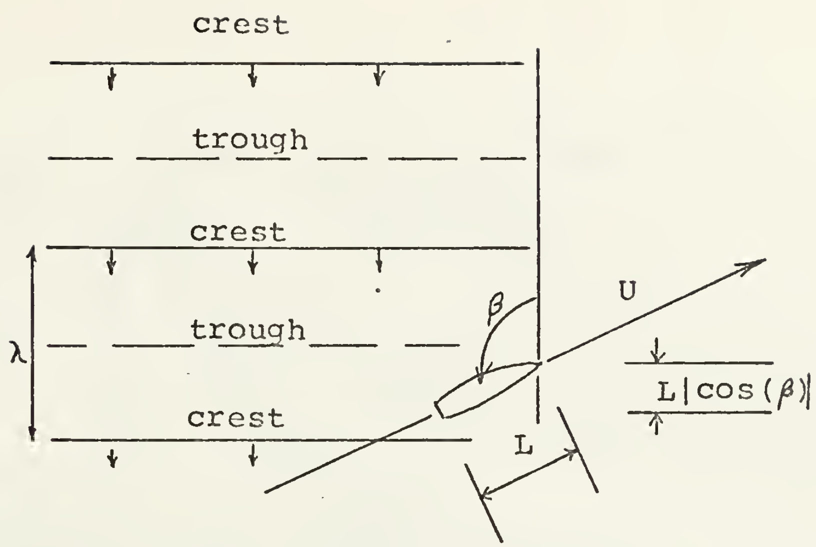


Figure 5. Ship in Long Crested Waves.

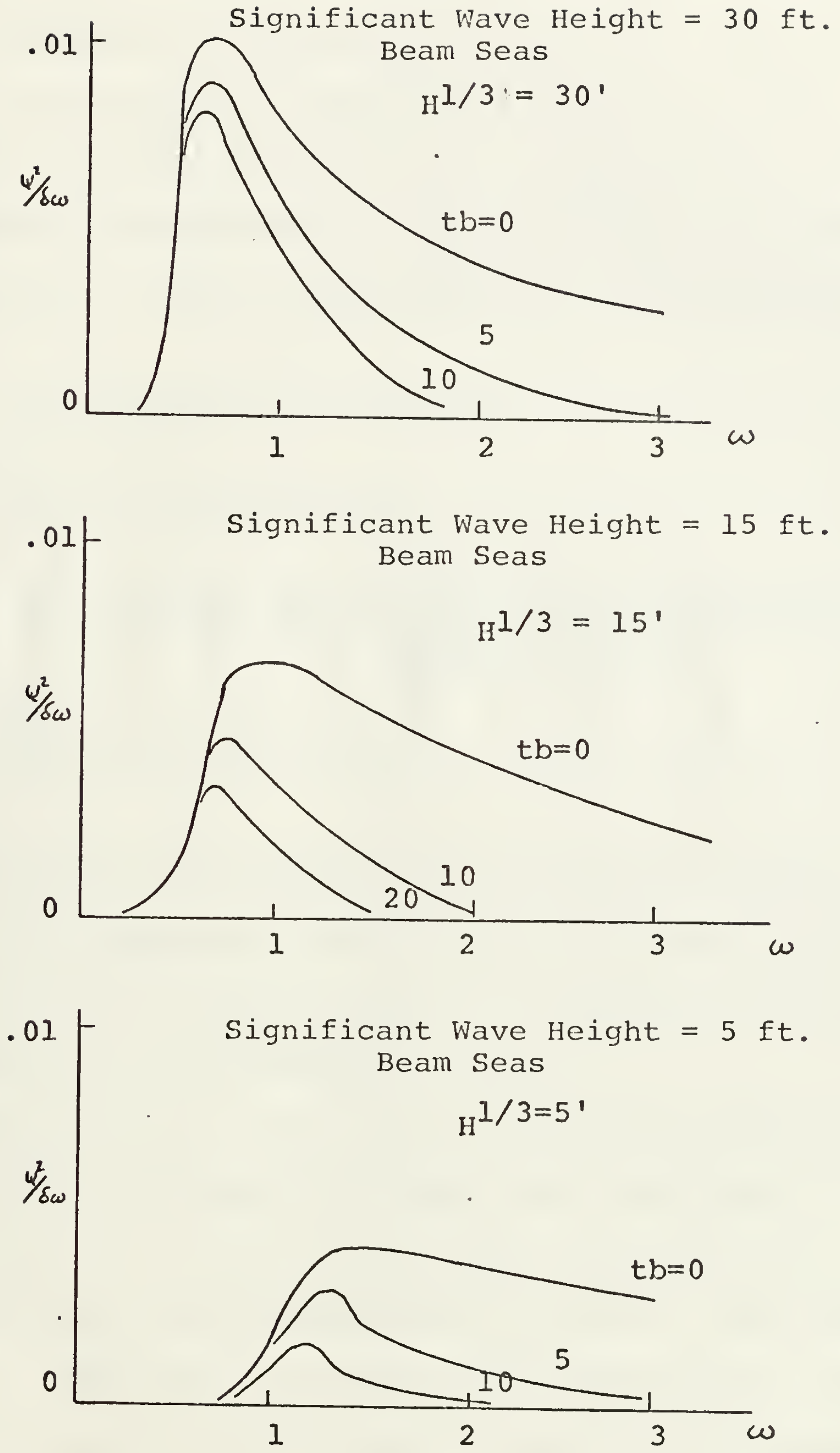


Figure 6. Excitation Spectral Density in Long-Crested Waves.

It seems reasonable to assume that the excitation at the center of buyoyancy is equivalent to that for the whole ship as long as the value of $L|\cos(\beta)|$ is less than a half-wavelength. Another consideration is that the roll excitation, the more significant of the excitations, is proportional to K_ϕ. A look at the terms in K_ϕ is enlightening.

$$K_\phi = \rho g(\text{Immersed Volume})\ GM$$

$$K_\phi = (\text{a constant})\ GM$$

$$BM = \frac{(\text{2nd moment of waterplane area about x-axis})}{(\text{Immersed Volume})}$$

$$GM = BM + KB - KG$$

The above implies that K_ϕ is proportional to the 2nd moment of the waterplane area. It can be shown that for a rectangle, the 2nd moment is $LB^3/12$. Therefore, it can be seen that K_ϕ is proportional to $(\text{beam})^3$.

Since the beam is not constant along the ship's length, only the wider portions of the ship will contribute to roll excitations. It is easy to calculate that any section with less than 60% of the maximum beam will have a negligible contribution. Therefore, the ship can be considered to have an effective length Le which should be used in the formula $L|\cos(\beta)|$ to replace the waterline length L. An empirical formula, which is a conservative estimate based upon the 60%

max. beam criterion, was derived graphically using a Taylor Series hullform from reference 9.

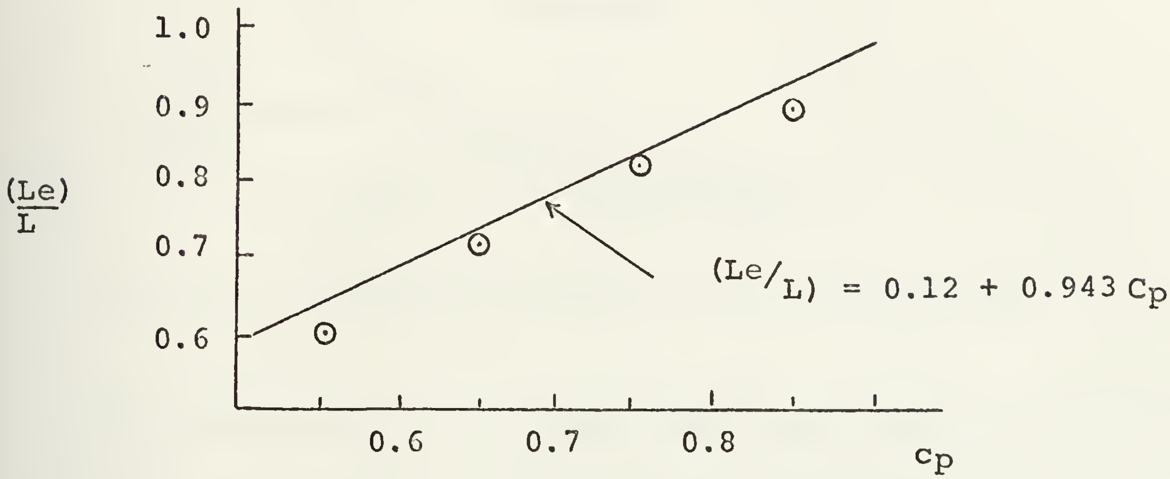


Figure 7. Effective Shiplength.

The region of validity for heading angle is therefore given by:

$$Le|\cos(\beta)| \leq \lambda/2$$

$$\text{but} \ldots \quad \lambda = 2\pi g/\omega^2$$

Applying this standard to graphical display is done in Figure 8 below.

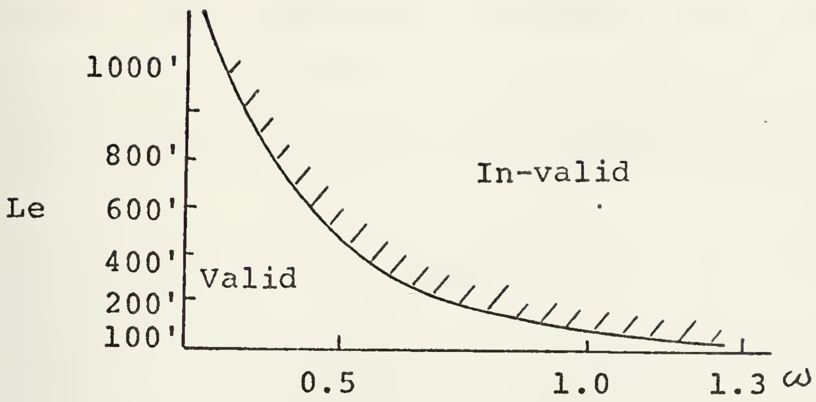


Figure 8. Region of Validity for Heading Angle.

To summarize, it is obvious that the range of parameter values for ω and β vary significantly with ship dimensions, ship geometry, and sea state. It is important in each case to investigate whether the extended equations are still valid.

7. Heading Angle Correction

From the results of Section 4, it would seem desirable to extend the range of heading angles further by some approximate means. An easy way to do this is with the assumption that the static righting moment is uniformly distributed along the effective length of the ship. Then, the wave excitation vanishes anytime that $Le|\cos(\beta)|/\lambda$ has an integer value of 1 or greater. It can also be surmised that for non-integer values of $Le|\cos(\beta)|/\lambda$ greater than 1, the part of the ship's effective length between successive crests receives a net zero excitation due to cancellation effects. It is appropriate to include this effect as a correction

factor to the excitation spectrum. An efficiency parameter, ε, which is a function of Le, β, and $\lambda(\omega)$ is formed by putting the above arguments into mathematical form.

let: $X = Le|\cos(\beta)|/\lambda$

X is between		$\varepsilon[Le, \beta, \lambda(\omega)]$
0.0	0.5	1.0
0.5	1.0	2(1 - X)
1.0	1.5	2/3(X - 1)
1.5	2.0	2/3(2 - X)
2.0	2.5	2/5(X - 2)
2.5	3.0	2/5(3 - X)
3.0	3.5	2/7(X - 3)
3.5	4.0	2/7(4 - X)
4.0	4.5	$\simeq 0$

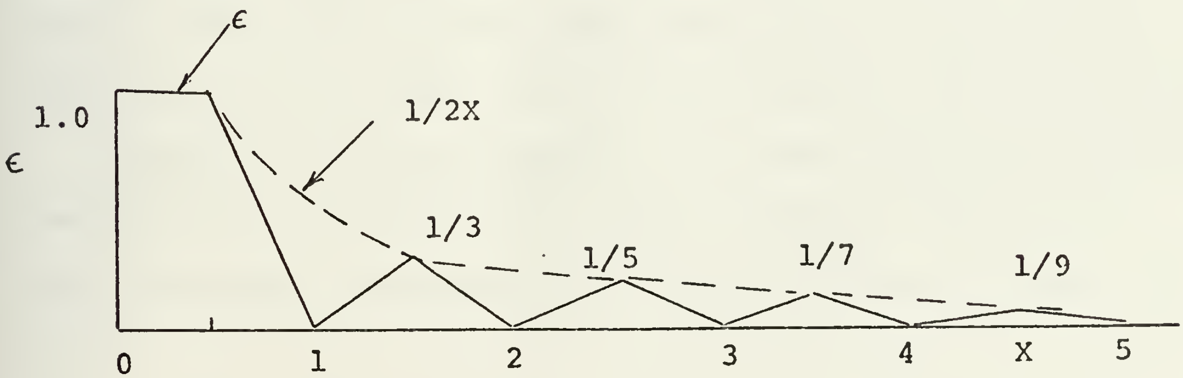


Figure 9. Heading Angle Correction.

The excitation spectral density function now is:

$$\psi^2/\delta\omega = \varepsilon^2 \sin^2(\beta)\, e^{-2tb\omega^2/g}\, S_{\xi\xi}(\omega)$$

Since an actual ship will probably not have a constant beam along its effective length, K_ϕ will not be uniformly distributed. The region of net zero excitation will not be quite equal to one wavelength, but will change with the relative orientation of the passing wave. Qualitatively, this will tend to introduce higher harmonics into the encounter frequency of the roll-inducing wave energy than that predicted by the above model. However, in general it is safe to assume that the cancellation effect does occur, even though the higher harmonics are produced.

Another consideration is that the final product of interest, roll angle, will be determined from integrating the spectral density function, and taking the square root of the result. This set of operations makes the roll angle very insensitive to the exact form of the heading angle correction.

Based on the above comments, it seemed consistent with a conservative approach to take the correction factor as the line connecting the peaks of the function shown in Figure 9.

$$\varepsilon = 1.0, \quad X \leq 0.5,$$

$$\varepsilon = 1/2X, \quad \text{otherwise.}$$

8. Spreading Function

A spreading function was used to convert uni-directional long-crested seas (implied by $S_{\xi\xi}$) into a more realistic short crested sea surface. Following reference 3, the spreading function $f(\mu)$ is a symetric function centered on the primary direction of the seas.

$$f(\mu) = 2/\pi \cos^2(\mu), \quad -\pi/2 \leq \mu \leq \pi/2$$

$$\int_{\pi/2}^{\pi/2} f(\mu) S_{\xi\xi}(\omega) d\mu = 1.0 S_{\xi\xi}(\omega)$$

The effect of this function is to smooth the long crested seas prediction for roll response somewhat. It implies that there is always a beam component of wave motion influencing the ship even when the ship is headed into or away from the predominant direction of the waves.

9. Sea State Data

The below listed values were taken from references 8, 11 and 12, and are useful for design purposes. The peak frequency of the surface wave elevation spectrum $S_{\xi\xi}$ is the ω_ρ given.

ω_ρ	$H^{1/3}$	Sea State	Probability of Occurence in North Atlantic
0.417	30	8	0.01
0.455	25	7+	0.02
0.509	20	7	0.03
0.589	15	6+	0.10
0.720	10	5	0.35
1.02	05	3	0.45
****	<05	0-2	0.04

IV. Solution for the Equations of Motion

1. Introduction

Given that the coefficients are all known, there are at least three practicable methods of solution available. These are analog, numerical integration, and complex-algebraic manipulation of the linearized equations. Each of these methods has its own advantages and drawbacks compared to the others. The latter method was the one finally adopted because it was the most flexible for design purposes and yields the most useful information for effort put into using it. However, further work with the other methods may be warranted depending upon the particular anti-rolling tank problem being investigated. The details of the complex algebraic method are given in this chapter. A brief overview of the results of the investigations into the other two methods is given in Appendix D.

2. Complex Algebraic Solution

The use of complex algebra along with the spectral density technique allows for a closed form solution of the linearized equations of motion. The basic assumption involved in this solution bears a close resemblance to what may be imagined as the real situation of the response of a ship in waves. It should be recalled that gravity waves have an advance velocity which is dependent upon frequency, so that

the phasing between ocean waves is essentially random. The waves of different frequency (or wavelength) travel along at different velocities. If it is assumed that the ocean has a constant spectral density with time, then the ship's response to each of these frequency components has reached a steady state value. Random phasing of these components accounts for the Rayleigh type distribution of peak values of response, and the principal of linear superposition neatly ties the whole system of equations and density functions together; the ship's response is the sum of the steady state responses to each of the frequency components.

A mention should be made of the implications of the term "linearity". The basic mathematical operation for each component case of frequency excitation and response is linear because the coefficients have been determined. However, the expressions for the so-called linear coefficients which involve ω_e, ω_o, X, phasing, ship dimensions and geometry, tank saturation, heading, sea state, etc. have no such restrictions. They are required only to be analytical.

The standard method for finding the steady state solution to a set of ordinary linear differential equations is to make a complex transformation, using a complex algebraic operator for differentiation, solve the complex equations algebraically, and retrieve the information of response magnitude and phase angle from the complex expression.

Let $\psi = \psi_o e^{i\mu\tau}$. Then the steady state solution must be of the form: $\phi = \phi_o e^{i\mu\tau}$, $\eta = \eta_o e^{i\mu\tau}$, $v^* = v^*_o e^{i\mu\tau}$, where ψ_o, ϕ_o, η_o, v_o^* are complex constants. The operator $D = d(\ )/d\tau$ may be replaced by $D = i\mu$, $D^2 = -\mu^2$. The equations of motion now become for a passive tank:

$$\begin{bmatrix} C11(i\mu) & C12(i\mu) & C13(i\mu) \\ C21(i\mu) & C22(i\mu) & C23(i\mu) \\ C31(i\mu) & C32(i\mu) & C33(i\mu) \end{bmatrix} \cdot \begin{bmatrix} \phi_o \\ v_o^* \\ \eta_o \end{bmatrix} = \begin{bmatrix} 1-\lambda_b\lambda_y+i\lambda_b\lambda_o \\ \lambda_y+i\lambda_o \\ 0 \end{bmatrix} \psi_o$$

$$\underline{C}(i\mu) \begin{bmatrix} \phi_o \\ v_o^* \\ \eta_o \end{bmatrix} = \begin{bmatrix} 1 - \lambda_b\lambda_y + i\lambda_b\lambda_o \\ \lambda_y + i\lambda_o \\ 0 \end{bmatrix} \psi$$

$$\left|\frac{\phi o}{\psi_o}\right| = \frac{\overline{\underline{N(i\mu)}}}{\text{determinant } [\overline{\underline{C(i\mu)}}]}$$

The numerator, $\overline{\underline{N(i\mu)}}$, is found by replacing the first column of the C matrix with the coefficient matrix for ψ_o. Notice that the C matrix terms will not be symmetric because of the choice of v* as a variable. If y* were chosen instead of v*, the symmetric condition for an energy conservative system would have been met.

Since $p = i\mu\phi$ by linear theory, the roll velocity response may be obtained by $\mu\left|\frac{\phi o}{\psi o}\right|$, and likewise the roll

acceleration is $\mu^2 |\frac{\phi_o}{\psi_o}|$. Tank angle, velocity and acceleration are similarly determined.

The final step in obtaining roll response is to multiply the transfer function ϕ^2/ψ^2 (or $\dot{\phi}^2/\psi^2$, or $\ddot{\phi}^2/\psi^2$) by the excitation spectral density function $\psi^2/\delta\omega$ and integrate the result over the entire frequency band. The spreading function $f(\mu)$ may be inserted in this step depending upon whether long-crested or short-crested sea result are desired.

$$\phi_{rms} = \sqrt{\int_0^{\infty} (\phi^2/\psi^2)\,\psi^2/\delta\omega \; d\omega} \qquad \text{(long-crested seas)}$$

$$\phi_{rms} = \int_{-\pi/2}^{\pi/2} f(\mu) \sqrt{\int_0^{\infty} (\phi^2/\psi^2)\,\psi^2/\delta\omega \; d\omega d\mu} \qquad \text{(short-crested seas)}$$

$$\phi^{1/3} = 2.0 \; \phi_{rms}$$

V. Anti-Rolling Tank Design

1. Introduction

The complex algebraic form of solution of the non-dimensional equations of motion was written in computer language to allow the numerical work to be done by a digital computer. Details of this computer program are given in Appendix C. Essentially there are two separate programs--MAIN 1 which maps $\phi^{1/3}$ versus μ_t and K_t for one case of heading angle in long-crested seas and MAIN 2 which calculates short-crested sea results for any given set of input parameters. In the interest of numerical efficiency most of the iterative steps of the design process can be done by MAIN 1, looking at the largest component of roll in long-crested seas which makes up the center frequency of short-crested seas results. MAIN 2 can be used later in the final stages to give short-crested sea roll angle predictions based upon the results of using MAIN 1.

The design method basically entails "bouncing" back and forth between MAIN 1 and MAIN 2 looking for optimum parameters of the non-dimensional equations. The steps are manually done, meaning that each step represents a separately submitted computer run. About 2 to 4 computer runs are required for a final design for each different anti-rolling tank.

The passive tank design method in section 2 suggests how to specify various parameters of the equations and presents a logical sequence to arrive at an approximate tank size. There is also a suggested iterative sequence to be used as a design aid. One of the fortunate results of the subsequent chapter on design examples is the collection of equations shown on Figure 12. It is recommended that this worksheet be kept as a tally sheet and guide when performing the iterative steps.

The active tank design method given in section 3 is mostly a discussion of possible changes to the passive tank design which are compatible with the computer program. Making these changes will result in a prediction of the performance of a feasible activated anti-rolling tank.

2. Passive Tanks

The response of the ship-passive tank system is modeled by the non-dimensional equations developed previously and setting $E_o(t) = 0$.

Tank location may be specified or a small number of different locations may be investigated. The reason for limiting tank location to the absolute minimum is entirely practical. The vertical space of the ship is divided by structure into a number of decks and for access and strength considerations the cross duct of the U tube tank should run across (or under) a deck. Also there may only be a single or a few

regions within the ship large enough to accept the tanks. As a consequence of the non-dimensional equations, there is a curious coupling between tank vertical placement, size, and the coefficients of the equations. Since the ship's vertical center of gravity is shifted with any vertical shift of the tank, the ship's GM is reduced whenever the tank is moved higher in the ship. Since K_ϕ is proportional to GM, and ω_s is proportional the square root of K_ϕ, and because all of the non-dimensional coefficients depend on K_ϕ or ω_s, each vertical tank location must be treated as a separate problem. The design method which follows is for each separate case of vertical tank location.

Once tank location is specified, the parameters which are still free to be chosen are: λ_t, μ_t, μ_{st}, K_t. We know that K_t has a minimum value of about -0.05, and that μ_t will have a value close to 1.0 because the vibration absorber effect of the tank is maximized by tuning the tank to $\mu_t = \omega_t/\omega_s = 1.0$. These four dimensionless parameters are dependent upon the real ship and tank parameters as follows:

$$\lambda_t = \lambda_t[K_\phi(GM), \rho A_o R^2, m]$$

$$\mu_t = \mu_t[\omega_s(K_\phi), R, S'(A_o/A_1, L_o/R)]$$

$$\mu_{st} = \mu_{st}[\omega_s(K\ ), R, S''(Z_t/R, L_o/R)]$$

$$K_t = K_t[\rho A_o R^2, \text{geometry of tank}, A_o/A_1]$$

If the ship changes its loading condition during expected operations such that its displacement is appreciably changed, then all four parameters have a Δ dependency, also.

Another important parameter for design is ψ max. Tank capacity ψ max is defined as the maximum heel angle that the tank can impart to the ship by maximum movement of the tank fluid off to one side. This also represents the maximum waveslope angle which the tank is capable of stabilizing. To form an opinion of a required ψ max, use must be made of empirical formulas. Hagen mentions the use of:

$$\psi_{max} = 0.36/\mathrm{Log}_{10}(\Delta) \text{ radians}$$

A formula suggested by center of buoyancy correction to the spectral density function which takes the equivalent of a 6° surface waveslope at the center of buoyancy for the ship's natural resonant frequency is:

$$\psi_{max} = 0.10\, e^{-(t_b \omega_s^2/g)} \text{ radians}$$

The above formulas give an idea of the approximate capacity which a tank must have in order to be effective, and serve as a feasibility check before proceeding further with a particular design.

The parameter $\lambda_t = (-2\rho g A_o R^2/-mgGM)$ has certain characteristics which allow it to be specified by the designer.

Rearranging the formula to read $\lambda_t = [2A_oR^2/(\text{displaced volume of ship})]/(GM)$, notice that the numerator of this ratio is the standard free surface correction for a virtual rise in the ship's center of gravity due to the fluid being unconfined in the η direction. Therefore, λ_t is the percentage loss in static stability of the ship due to the tank fluid free surface. There will be a maximum allowable λ_t which ensures safe operation of the ship under all loading conditions, including damaged conditions. Since a larger λ_t is associated with more tank stabilizing effectiveness ($\psi_{max} \simeq \lambda_t L_o/R$), the choice of λ_t is fixed at the maximum value which allows safe operation of the ship.

An important conclusion to reach from the above observations is that the ship hull form should be designed with the notion that an anti-rolling tank is to be installed. This means that space, weight, and static stability will be such that the ship will be comfortable with a tank of the capacity indicated by the empirical formulas. The alternative choice of trying to fit an anti-rolling tank into an existing ship design which may not have all the necessary reserve space, weight, or static stability most likely will result in an ineffective stabilizing system.

Now that λ_t is known, and ψ_{max} has been determined approximately, an assumption for a μ_t value is all that is required to calculate tank dimensions (A_o, R, L_o, z_t) the tank parameter $\mu_{st}(L_o/R, z_t/R)$, and approximate tank weight

$[2\rho A_o R(L_o/r + A_1/A_o)]$. A first guess at μ_t is made by using Chadwick & Klotter's equation, based upon the linear equations, neglecting sway, from reference 4.

$$\mu_t = [1 - \lambda_t(1/\mu_{st}^2 - 1/\mu_{st}^4)]^{-1/2}$$

It is now possible to calculate all the tank dimensions and all the coefficients of the equations of motion with sufficient accuracy to consider them constant, except for duct area A_1 and tank parameters μ_t and K_t.

The remainder of the design problem may be stated as follows: Find the best combination of μ_t and K_t which minimizes the rolling motion predicted by the non-dimensional equations of motion. This two parameter search problem may be solved by noticing that μ_t must be about 1.0, and that two digit accuracy is the best to be expected from the equations of motion. Therefore, instead of a sophisticated search technique it should suffice to map roll angle versus μ_t and K_t. This may be further simplified by sub-optimizing each μ_t value for K_t. This approach is justified by gentle curvature of the function $\phi^{1/3}(\mu_t, K_t)$, in the region of interest, as found in all sample problems tried. The remainder of this section will investigate numerical methods to deal with this search problem.

The basic strategy for solving this problem is to first fix upon some criterion of roll performance. Based upon the sea

state data already given, the author decided to minimize roll angle at the worst heading at operational speed in sea state 6 (15 feet significant wave height) was a reasonable criterion. Other criteria could include roll velocity or roll acceleration for other environmental conditions.

In order to compute $\phi^{1/3}$ for given conditions, a long sequence of operations is accomplished--$\phi^{1/3}$ is twice the square root of the integral of ϕ^2/ψ^2 times $\psi^2/\delta\omega$. This sequence makes vivid the reason why the complex algebraic method of finding ϕ/ψ is superior to numerical integration. The latter would require another integration of many steps, a very expensive approach since the results must in turn be iterated many times for a good design.

The above discussion assumed that long crested seas were involved. For short crested seas another integration, this time over every possible heading angle, must be performed for the spreading function. For this reason, long crested seas are assumed for mapping $\phi^{1/3}[\mu_t, K_t(\text{best})]$. Short crested seas are only looked at later as an output.

In mapping $\phi^{1/3}[\mu_t, K_t(\text{best})]$, it was found that the function displays very gradual curvature as was to be expected for an integral function. Therefore, only a few points are investigated during each computer run, because interpolation is predictable. A second order Legendre polynomial curve is used in an interpolation scheme to find a local minimum or maximum. Manual inspection of the computer

results will show whether further mapping is necessary. Warning: the interpolation function likes to extrapolate to find minimums or maximums equally well, but such results are invalid by the statement of the problem.

Once $\phi^{1/3}[\mu_t, K_t(\text{best})]$ has been mapped and values of μ_t and K_t for a minimum roll angle have been decided, short-crested sea results may be obtained. The problem then shifts to chosing which of the alternative tank locations is the best. Economic as well as performance factors will enter into the decision of this question. For example, a tank which is in a cargo hold, presumably valuable real estate, would probably not be chosen over one which was in a less costly part of the ship, such as atop an outside deck, were the two tanks to perform nearly as well.

A detailed listing of the author's steps in passive tank design are reported as an aid in improving upon them. A later section will include a completely worked example of the method.

I. Problem setup

a) Assume a range of a small number of vertical tank locations.

b) Calculate ψ_{max} requirement.

c) Specify a λ_t

d) Assume an initial μ_t value from Chadwick and Klotter's equation.

e) Determine Lo, A_o, R, z_t, and approximate A_1.

f) Design the tank except for A_1.

g) Calculate the hydrodynamic coefficients and other computer inputs.

II. First Iteration

a) Run MAIN 2 program; use the long crested wave results to estimate the worst speed and heading angle combination. Use μ_t assumed, $K_t = -0.8$

b) Map $\phi^{1/3}[\mu_t, K_t(\text{best})]$ using MAIN 1. Use μ_t assumed, K_t initial $= -0.4$, $\Delta\mu_t = 0.5$, $\Delta K_t = -1.0$.

c) Interpolate to find $\phi^{1/3}$ minimum. Check computer results to ensure that a true minimum occurred.

III. Second Iteration

a) Run MAIN 2 again to see if the worst speed and heading angle combination has changed. Also, roll performance may have degenerated at other headings and speeds significantly.

b) Continue iterations if change in speed or heading angle warrant it.

IV. Alternative Tank Choice

a) Based upon the operator's or owner's criteria, select the best tank location.

V. Verification

As a final step, short crested sea results, MAIN 2, should be run for the entire expected range of speed and sea conditions. This is to make sure that the solution didn't fix too shortsightedly upon some assumed worst condition and lessen it without overlooking its negative effects on some peculiar or unforseen combination. Another verification which should be performed during this and all other steps as well is to check the range of parameters, the range of the solution variables to ensure that their coefficients remain linear (or else adjust the coefficients to their equivalent linear values) and that the μ_t and K_t values being predicted are realizeable.

3. Activitated Anti-Rolling Tanks

An activated anti-rolling tank is one which has an input power source in addition to ship induced excitations. A pressure source is used as the type of power input in this discussion, as opposed to a flow source. It is desired to regulate the pressure source in such a way as to reduce the rolling motion of the ship.

The choice of a pressure source instead of a flow source was not entirely arbitrary. A flow source would have to be some sort of a positive displacement pump capable of handling large volume rates of fluid transfer, approximately 500 tons per minute as an order of magnitude. The required pump size

for this volumetric capacity would make it economically unfeasible. Another requirement for the power source that is imposed by economics and amply illustrated in other papers, e.g. reference 18, is that it operate at a nearly steady power level. The peak power output for a pressure source, as found by standard control theory methods using linear feedback, is too high to be practical. There is too much wasted power capacity which is only used at peak power demand. However, the pressure pump must be built large enough to be able to supply the predicted peak demand if the linear equations are to be valid. Once further economic factor to be considered is the competition, the alternative methods of roll damping. With regards to these, the gyroscopically controlled anti-rolling fin installation is the most serious candidate. In practice, a standard, well designed anti-rolling fin installation has been found to consume about one or two percent of the installed hoursepower of a ship, including propulsion, due to the added effects of the fin area increasing the ship's total hull resistance to passage through the water and the power needed by the fin positioning motors. Thus, economics and practical engineering considerations have dictated a rather narrow range of possible control effectors for anti-rolling tanks--a pressure type source of power of nearly constant output which has a power rating of about one percent of the ship's installed power.

A further consideration of the power source is reversibility. A level power output does not imply that the direction of pressure or flow will be constant. The pressure supply will obviously have to reverse itself as the ship rolls back and forth. This aspect of the pressure source, quantified by a characteristic time lag to change direction, will have to be reasonably estimated and included in the analysis, and must also be reasonable in the sense that a smaller time lag probably represents a more expensive piece of equipment.

It is helpful to first look at the orders of magnitude of the parameters needed to describe a control effector. Consider a ship of 20,000 tons displacement with a speed of 20 knots. Assuming an overall lift-to-drag ratio of 150, the installed horsepower would be about 18,400 Hp. This would indicate from the preceeding arguments that an economically feasible anti-rolling tank can be no greater than 184 Hp. Compare this to the energy flow of the ship in its roll coordinate, assuming a GM of 6', a 5° roll, and a roll period of 12 seconds.

$$20{,}000 \text{ tons } (2240 \text{ lb/ton})(6')(\sin 5°)(1/2) \cdot$$
$$(1/12 \text{ sec})(\text{Hp-sec}/550 \text{ ft-lb}) \;=\; 1775 \text{ Hp}$$

This simple estimate is enough to show that we cannot expect the control effector to be able to counter the effects of wave excitations. However, it may be used to counter the

resonance phenomenon associated with rolling frequencies near the natural resonance peaks of the ship-tank system.

Since the passive anti-rolling tank is capable of developing nearly as much energy flow as the ship, until the saturation angle of the tank is reached, it seems justifiable to further simplify the control effector equation to

$$T_p \dot{p}(t) + p(t) = \pm P_c$$

P_c denotes the steady pump pressure developed across the tank duct. and T_p is the reversing lag time for the pump. The equation $P_c = f(\phi,\dot{\phi},\eta,\dot{\eta})$ implies now that the sign for P_c is chosen according to some criterion contained in the state variables $\phi,\dot{\phi},\eta$, and $\dot{\eta}$. Consider the same ship with an anti-rolling tank of 200 tons weight of fluid. The average fluid volume flow would be:

$$0.707(200 \text{ tons})(35 \text{ ft}^3/\text{ton } H_2O)(1/12 \text{ sec}) = 142 \text{ ft}^3/\text{sec}$$

For an average 184 Hp, the mean pressure is 184 Hp (550 ft-lb/Hp-sec)(1/142 ft^3/sec) = 245 lb/ft^2 = 1.7 psi. This pressure changes the fluid level between the tanks by 3.8 ft. Assuming a 50 percent pump efficiency, the pressure differential would be 0.85 psi with a fluid level difference of 1.9 ft. This corresponds to a tank angle of about two degrees. The static ship roll angle for still water and constant pump

pressure would be λ_t times two degrees, amounting to only a fraction of a degree. Thus we can now state that the addition of such a control effector would have a qualitatively negligible effect on the equations of motion, except near the resonant frequencies. This indicates that some form of active damping is the goal of the control effector switching criterion.

The problem of adding a control effector now becomes one of how to add the active damping to the linearized equations of motion. The tank equation is the only one of the set which contains p(t) for an active tank. Substituting $P_c = g_1\phi + g_2\dot{\phi} + g_3\eta + g_4\dot{\eta}$ into the tank equation, neglecting for the moment T_p (consider T_p small), and combining terms in the linearized, non-dimensional equations of motion, the structure of the active tank equations is revealed.

$$\lambda_t[D^2/\mu_{st}{}^2 - \frac{\omega_s g_2}{2\rho g R}D + (1 - \frac{g_1}{2\rho g R})]\phi - \frac{\lambda t D^2}{\mu_{yt}{}^2}\,\nu^*$$

$$+ \lambda_t[D^2/\mu_t{}^2 + (\frac{K_t}{\mu_t} - \frac{\omega_s g_4}{2\rho g R})D + (1 - \frac{\omega s g_3}{2\rho g R})]\eta \;=\; 0.$$

We would expect that the effects of g_1 and g_3 to be relatively small even if we took them at their maximum possible values (to develop 184 Hp) because of the small static values of tank and roll angle the pump is able to develop. Hence, making $g_1 = g_3 = 0$ is approximately correct for any

economical tank. The choice of g_2 and g_4 to use as a switching criterion is obvious. The tank already has linear damping which may be varied as required by adding valves or baffles. Negative tank damping is undesirable because it would increase roll response at resonance frequencies. The computer program results from MAIN 1 confirms this hypothesis. Adding a damping term to the ship roll-tank coupling terms, however, would have a desirable effect at any frequency. An additional tank moment caused by the pressure differential would be induced opposing the roll velocity of the ship. Thus, the controller effects may be included in the linearized equations of motion for the type of controller described by substituting this reduced tank equation into the nondimensional equations of motion.

$$\lambda_t [D^2/\mu_{st}{}^2 - \frac{\omega_s g_2}{2\rho g R} D + 1] - \frac{\lambda_t D^2}{\mu_{yt}{}^2} \nu^*$$

$$+ \lambda_t [\frac{D^2}{\mu_t{}^2} + \frac{K_t D}{\mu_t} + 1]\eta = 0$$

The value of g_2 is determined by power considerations. The energy dissipation, the product of assumed pump pressure and tank volume flow, can be calculated and compared to the desired pump power in an iterative fashion until the two balance. The effect of pump lag time may be included by deducting the fraction of the period of roll that was spent

in changing direction. This factor is about $2\pi/\omega - T_p$ for T_p less than $2\pi/\omega$ and zero for T_p greater than $2\pi/\omega$.

The above active tank equations are only useful for predicting ship motions in a seaway which overpowers the passive aspects of the anti-rolling tank. Fortunately, this is the case when an active tank would most likely be used. The additional damping term contained in g_2 and the need to calculate mean pump power are small modifications to the computer program for the equations of motion. Otherwise, the same design method as that for the case of passive anti-rolling may be used.

The type of hardware which would give the desired pump characteristics is a waterjet type pump situated in the tank cross duct. The jet velocity would be much higher than the mean fluid velocity, giving rise to a steady pressure independent of average tank fluid velocity. The presure rise is caused by the cross duct acting as an inefficient diffuser and the pressure reverse may be accomplished by switching the waterjet outlet from a nozzle in one direction of flow to an opposite nozzle. The actual pump motor would then not have to reverse at all, and the lag time would be associated with the time required to switch nozzles.

VI. Design Examples

In order to validate the computer program, verify the design steps postulated, and investigate the possible range of the solutions, it was decided to design a series of antirolling tanks for a sample family of ships. The ships were chosen to be able to illustrate as many aspects and implications of the equations of motion as possible within time and effort constraints.

Three ships were finally settled upon to show basically the effects of ship size, degree of sway coupling, and tank size. They were a 3600-ton small naval escort type, a 14,500-ton medium sized naval vessel which resembles a small aircraft carrier or amphibious landing ship, and a 20,400-ton commercial stores carrier. The three ships were assumed to be existing designs to which an antirolling passive tank was to be added. The two naval vessels were possessed of a large degree of sway coupling introduced by the parameter $\lambda b = BG/B$ because they were ships with high centers of gravity relative to their other dimensions. The commerical stores carrier had only about half as much sway coupling, being more typical of a cargo vessel which does not possess the high topside payload weights of a naval vessel.

1. Ship Size

The principal dimensions of the three vessels were as follows:

	Ship No. 1	Ship No. 2	Ship No. 3
Δ	3600	14,500	20,400
L	408	585	500

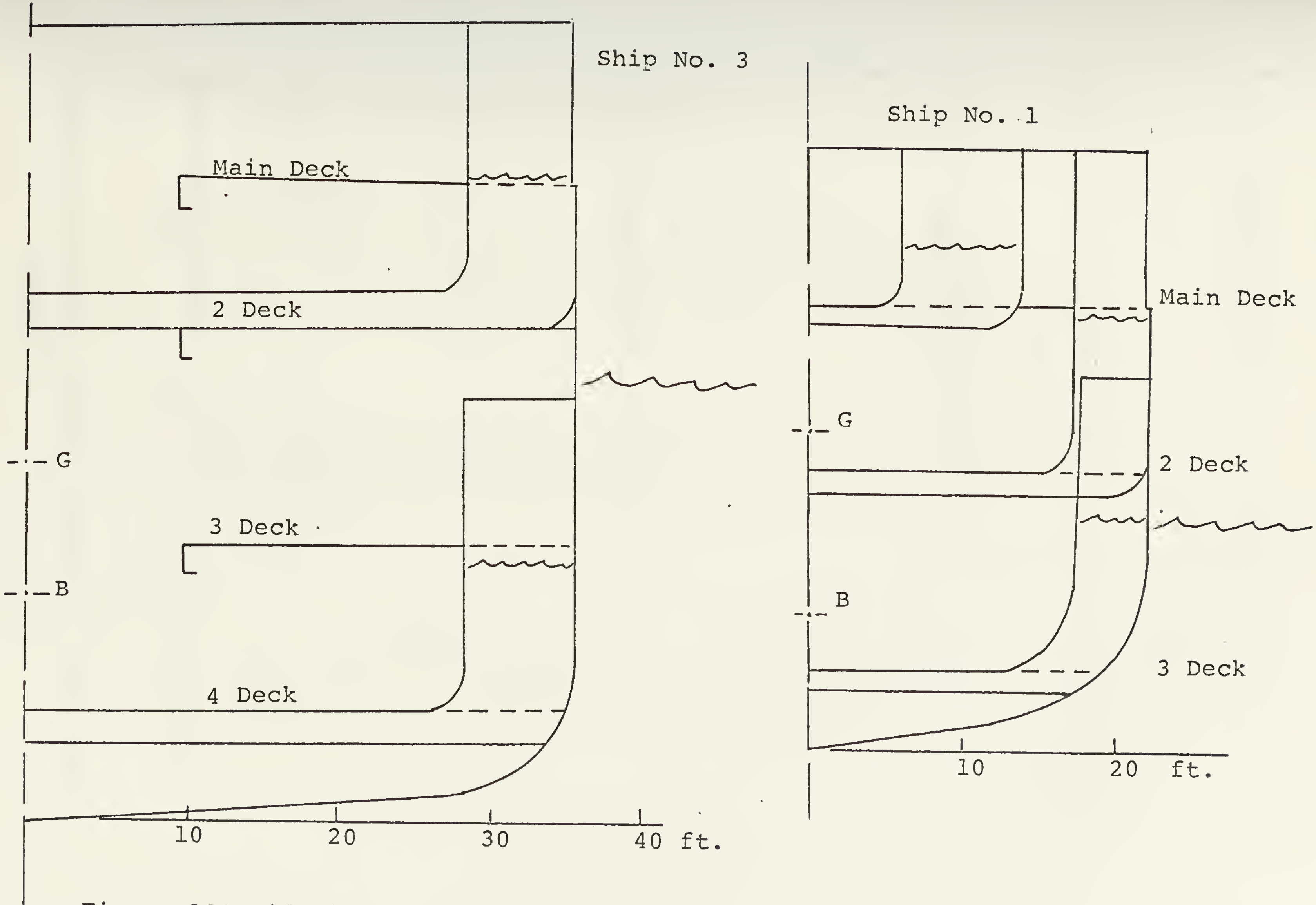


Figure 10A Midship Section Ship No. 1 and 3, With Tank Locations

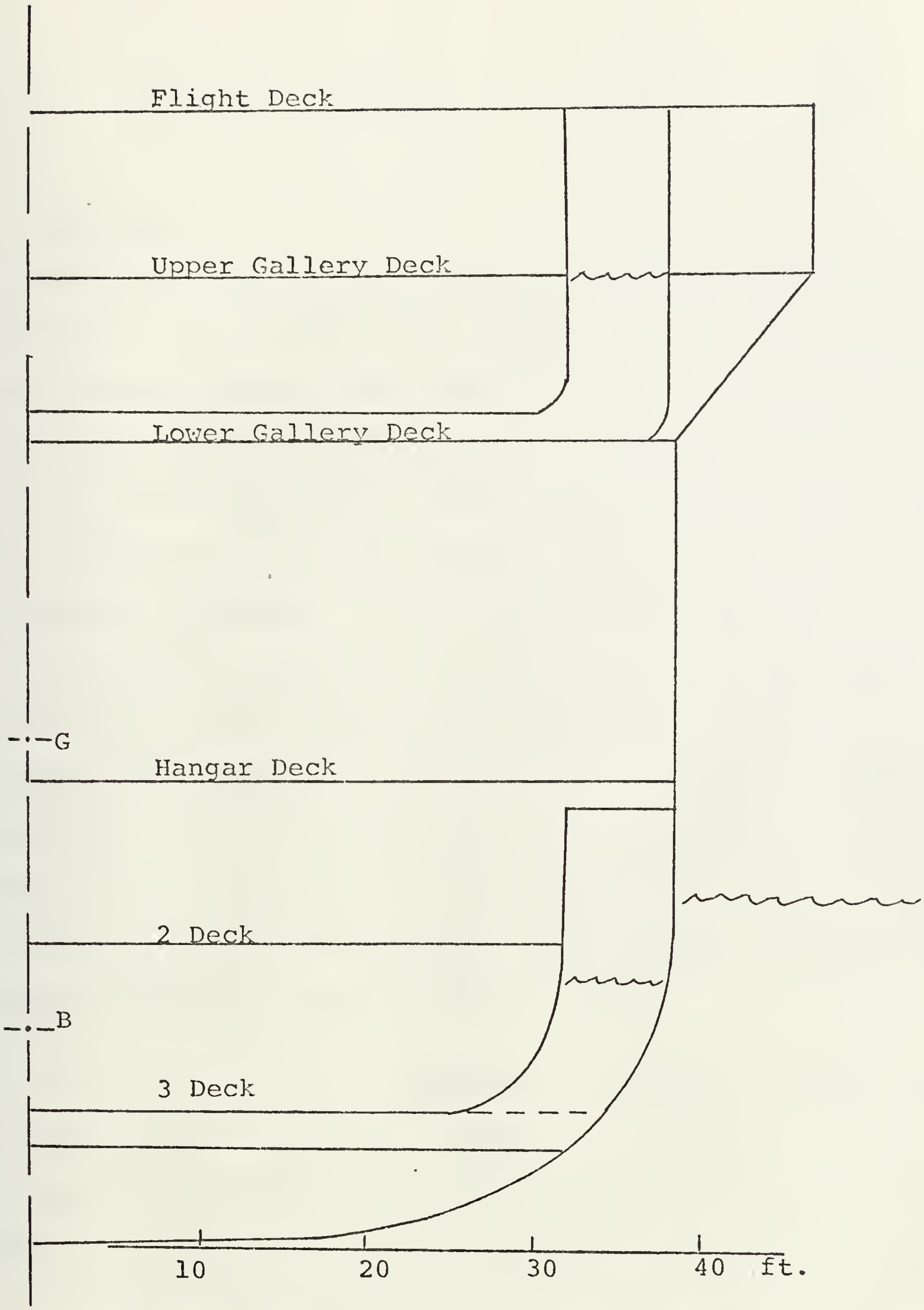


Figure 10B Midship Section Ship No. 2, With Tank Locations

	Ship No. 1	Ship No. 2	Ship No. 3
B	45	76.67	71.4
T	14.5	21.33	28.6
Cb	0.4733	0.5304	0.70

2. Tank Size

Based upon the empirical formula of Section V.2, an approximate required value for tank capacity, Ψmax., was determined. The vertical layout of the ships, by decks, is shown in Figure 10. Tank dimensions and parameters were calculated using the collection of equations shown on figure 12 by an iterative process. The λ_t value for all tanks was maximized in the following way: An assumed λ_t was used for the low tank and applied to the value of metacentric height, GM, for the low tank. The residual stability of the reduced metacentric height given by GM $(1 - \lambda_t)$ was kept as a required value to be achieved by the other tank locations. Since the higher tank locations resulted in a higher center of gravity for the ship and therefore lower GM, their λ_t values had to be necessarily lower in order to achieve the required residual stability GM $(1 - \lambda_t)$.

	Ship No.1	Ship No. 2	Ship No. 3
Ψ max. = $.36/\log_{10} \Delta$	5.8°	5.0°	4.8°
Ψ max. = $.1e^{-tb\omega_s^2/g}$	5.4°	5.4°	5.2°
GM $(1 - \lambda_t)$	2.69	5.17	5.25

Of the three deck locations possible for tanks on Ship No. 1, it was found that the tank placed on the main deck had too small

a capacity Ψ max. and ship-tank coupling λ_t to be practical. Only the lower tank and the center tank were further investigated.

On Ship No. 2 there were only two tank locations possible, assuming that keeping the center hangar deck area clear was imperative. The higher tank was slightly cramped in order to fit it between decks, while the lower tank should probably be expanded to meet the hangar deck bottom to eliminate the small wasted space between the tank top and the hangar deck.

On Ship No. 3, the presence of vertical cargo access hatches and cargo storage limited again the tank location to two possibilities: a low tank with the cross duct underneath the cargo floor, or to a second deck high tank of small dimensions to accommodate the cross duct between cargo hatches.

For the naval vessels, Ships No. 1 and 2, the tank fluid was assumed to be fuel oil such as diesel oil or JP-5 (ρ = 1.69 slugs/ft.^{3}) because such vessels carry a generous reserve fuel supply for extended range purposes. In effect, this made the passive antirolling tanks a reserve fuel oil tank which is possible to ballast with seawater in an emergency. For the commercial vessel, Ship No. 3, the tank weight was too large to consider it to be carrying expensive fuel oil, so seawater (ρ = 1.99 slugs/ft.^{3}) was assumed to be the tank fluid. This reduced the tank volume somewhat, compared with a fuel filled tank, but the tank weights of between 1 - 3 percent of the ship's total displacement was a factor against their being a practical commercial installation.

3. Operational Criteria

The choice of speed and sea state was also made to illustrate the range of solutions of the equations of motion. The smallest ship was examined in a sea state 5 ($H^{1/3}$ = 10 ft.) at its operational speed of 25 knots, a typical requirement for an escort ship of its class. Ship No. 2 was looked at in sea state 6 ($H^{1/3}$ = 15 ft.) at a lower speed of 20 knots because it can be expected to behave better than its lighter counterpart, and has a lower designed speed. Ship No. 3 was examined at its economical cruising speed of 16 knots in a storm of sea state 7 (20 ft. $H^{1/3}$). This is because the tank should be able to mitigate motion effects on the cargo in extreme conditions.

4. Initial Program Values

An initial estimate of the worst heading angle, the point for optimizing roll angle versus μ_t and K_t, was made by taking the point where the peak of the excitation spectral frequency distribution corresponded to the higher frequency peak of the roll response, assumed about 130 percent of ω_s. Later work showed that this was not consistently a very good first guess, and that a better first estimate could be gotten by running Main 2 program with Kt = -0.8 and taking the worst heading angle directly from the results.

The initial values for heading angle and other parameters for the first iteration were as follows for each of the ship-tank combinations.

	Ship No. 1		Ship No. 2		Ship No. 3		
	low	center	low	high	low	high	
BG	9.75	9.93	16.5	17.3	8.37	8.69	
GM	3.36	3.18	7.39	6.67	7.00	6.68	
t_b	5.67	5.67	8.24	8.24	13.43	13.43	
λt	.20	.155	.30	.22	.25	0.075	
μt	1.01	1.01	1.02	.97	1.01	1.00	
$\mu s t^2$	4.11	4.10	4.72	-24.61	4.25	-88.3	
Kp	-.221	-.221	-2.92	-2.92	-3.17	-3.17	($\times 10^7$)
$\lvert Y_{\dot{v}}/m \rvert$	.67	.67	.53	.53	.88	.88	
$\lvert K_{\dot{p}}/Ix \rvert$	.26	.26	.41	.41	.15	.15	
D1	-8250	-8250	-30200	-30200	0	0	
D2	-21450	-21450	-78500	-78500	-96790	-96790	
D3	-10300	-10300	-37700	-37700	-26800	-26800	
F1	0	0	0	0	0	0	
F2	.684	.684	.684	.684	.735	.735	
F3	1.05	1.05	1.05	1.05	1.13	1.13	
β	90°	90°	118°	118°	50°	50°	
U	43	43	34	34	27	27	
$H^{1/3}$	10	10	15	15	20	20	
KG	18.58	18.76	30.4	31.1	23.54	23.86	
$K\phi$	-2.71	-2.57	-24.0	-21.66	-31.99	-30.52	($\times 10^7$)
ω_s	.5413	.5271	.4456	.4233	.505	.493	
Ao	124.5	91.3	607	357	611	214	
Ao/A_1	5.0	5.26	4.61	4.85	3.4	3.83	
R	20	20	33	35	32	32	
Lo/R	.498	.538	.303	.286	.349	.313	

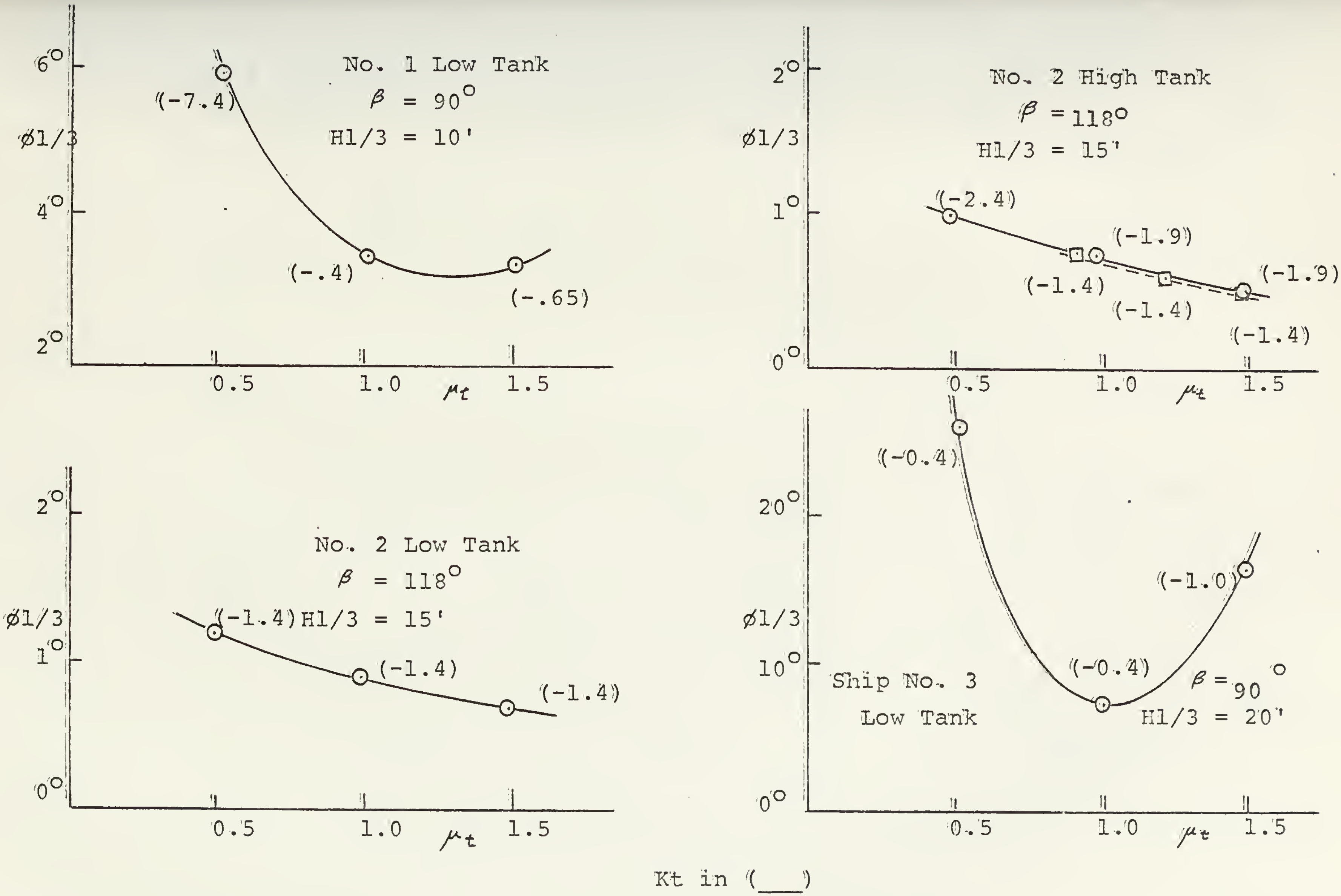


Figure 11A First Iteration Tank Optimization

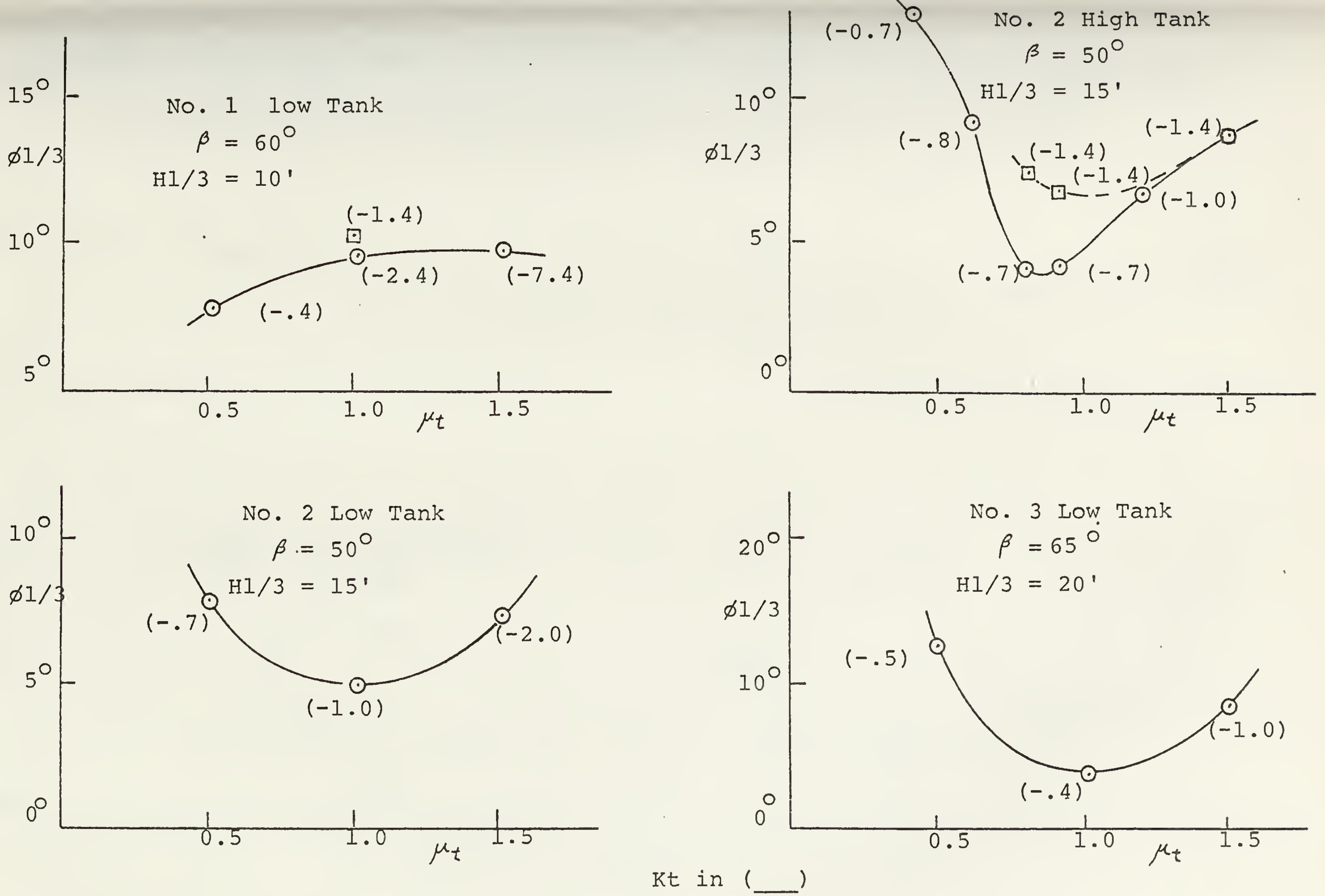
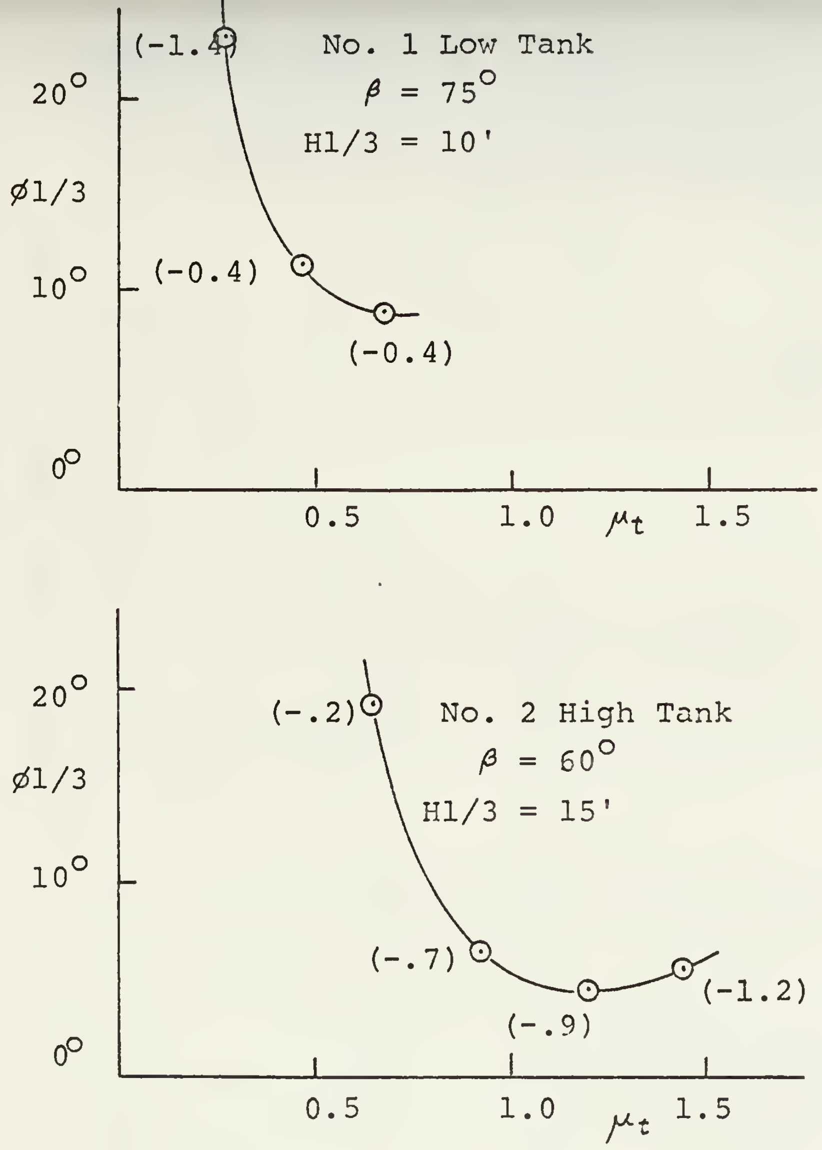


Figure 11B Second Iteration Tank Optimization



Kt in (___)

Figure 11C Third Iteration Tank Optimization

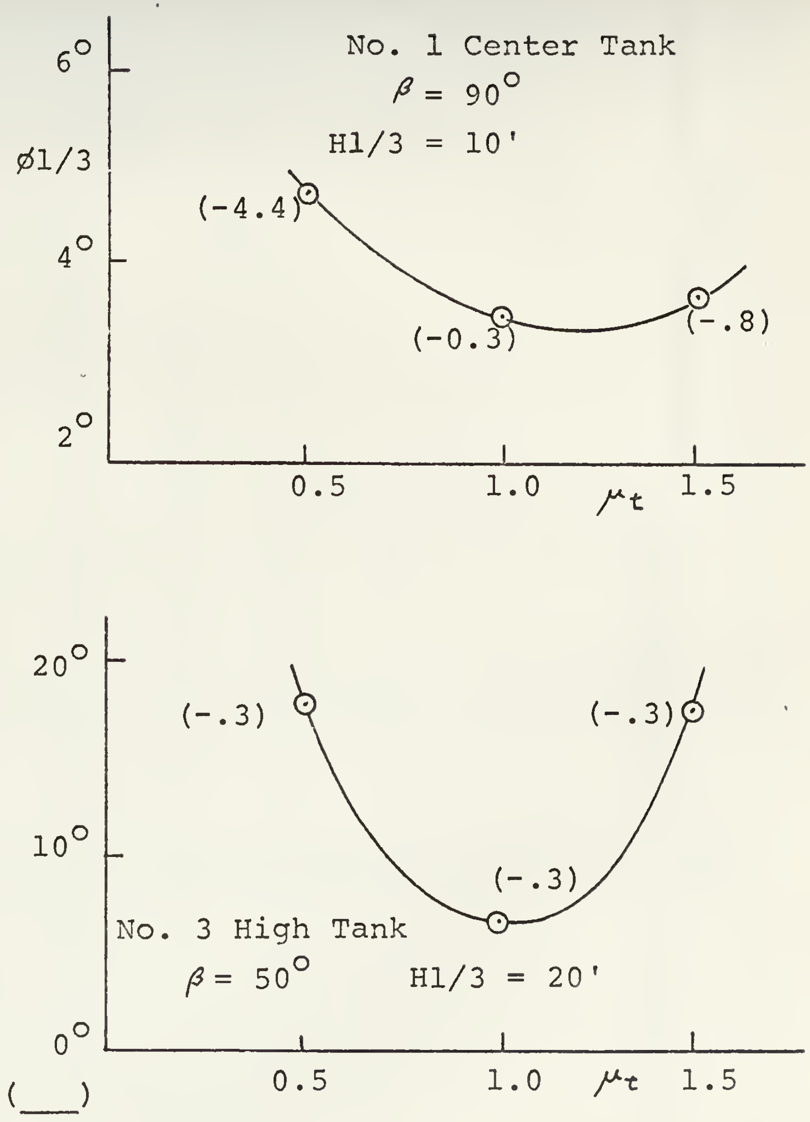


Figure 11D First Iteration Tank Optimization

SHIP:	TANK:			
	_iteration	_iteration	_iteration	_iteratior
$\psi_{max} = \lambda t \; Lo/R$				
$K_\phi = -mg \times GM$				
$\lambda t = \dfrac{-2\rho g A_o R^2}{K_\phi}$				
$Lo = (R/\lambda t)\; \psi_{max}$				
$A_o = \dfrac{\lambda_t K_\phi}{-2\rho g R^2}$				
$t \quad \sqrt{\dfrac{1}{1-\lambda t(\frac{1}{\mu_{st}^2} - \frac{1}{\mu_{st}^4})}}$				
$\omega_s = \sqrt{\dfrac{-K_\phi}{m(.38B)^2(1+\lvert\frac{Kp}{Ix}\rvert)}}$				
$\omega_t = \mu_t \omega_s$				
$A_o/A_1 = \dfrac{1}{\omega t^2 R/g} \; -Lo/R$				
$\omega_{st} = \sqrt{\dfrac{g}{R}\, \dfrac{1}{2Lo/R + Z_t/R}}$				
$\mu_{st} = \omega_{st}/\omega_s$				
$W_t = \dfrac{2\rho g A_o R}{2240}\,(L_o/R + A_1/A_o)$				
$KG' = (\Delta KG + \delta W \delta z)/\Delta$				
$GM' = KG + GM - KG'$				

Figure 12. Tank Sizing Equations - Worksheet

	Ship No. 1		Ship No. 2		Ship No. 3		
	low	center	low	high	low	high	
Weight	84	64.6	506	299	611	163	(tons)
z_t/R	.343	-.339	.436	-.78	.23	-.67	
ω_{st}	1.097	1.067	.968	i2.1	1.041	i4.75	

5. <u>Summary of Results</u>

The results of calculations for $\phi^{1/3}$ as a function of μ_t at K_t (best) are shown on figure 13 for the first three iterative steps of looking at an assumed point in the sea state - heading angle space. A discussion of results for each of the three ships follows this. It should be repeated that the intent was not to design a particular antirolling tank or to apply any arbitrary criteria, but instead to get as broad a look as possible at the range and implications of the equations of motion, and the solution method.

Because the center tank of Ship No. 1 did not seem to offer any better performance than the low tank, it was not carried beyond the first iteration. This low tank in Ship No. 1 became the most difficult of the tanks to design because the plot of $\phi^{1/3}$ versus μ_t at K_t (best) exhibits more curvature and other surprises than the other tanks. At each iteration, the sub-optimized parameters which worked well at that particular heading angle were poor at different headings. Finally, μ_t was chosen as 1.01 by inspection of all the data, and K_t was determined by running Main 2 program for short crested seas through $K_t = -0.4$, -0.8, and -1.0. The combination $\mu_t = 1.01$, $K_t = -0.8$,

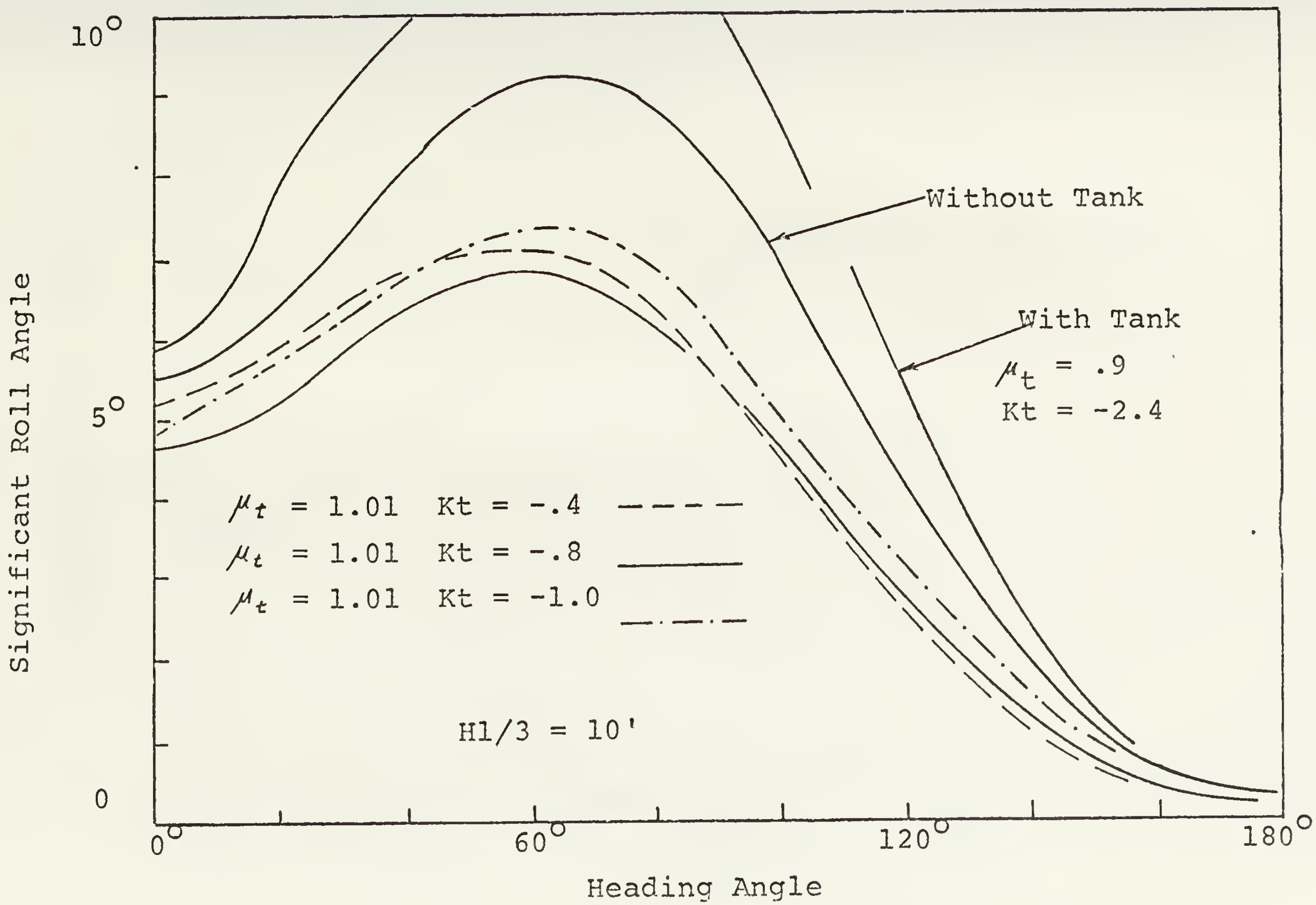


Figure 13 Ship No. 1 Low Tank

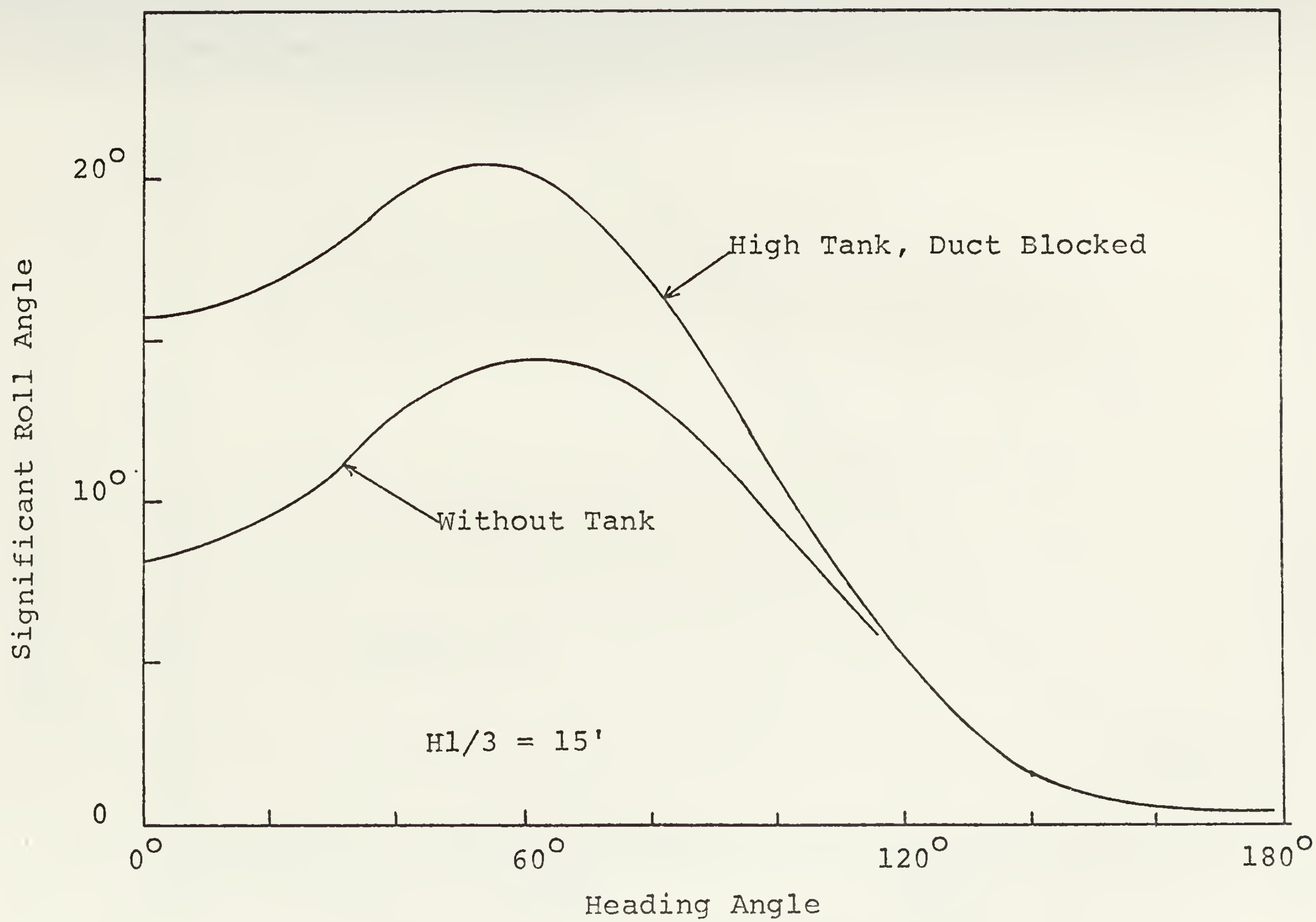


Figure 14A Ship No. 2

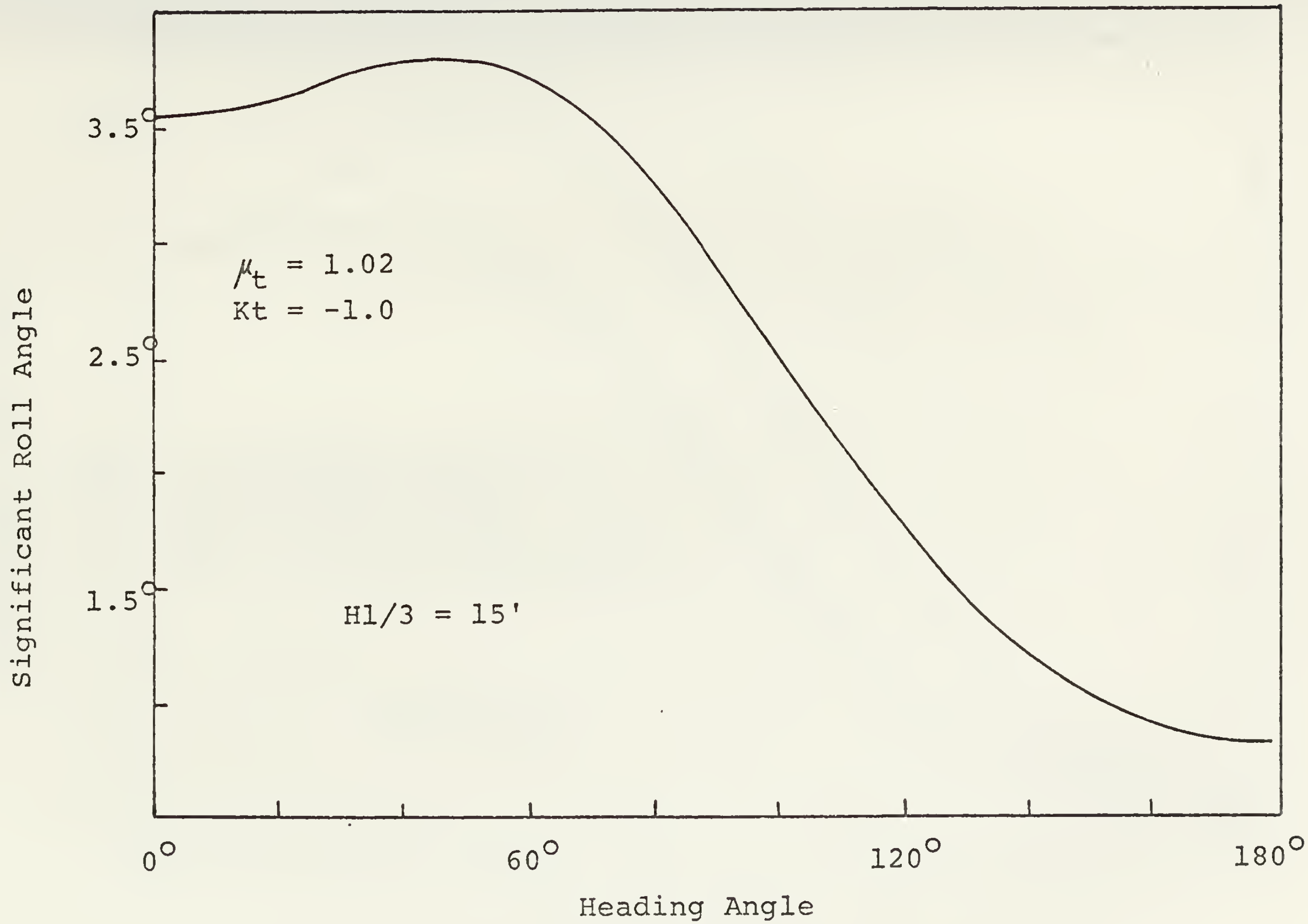


Figure 14B Ship No. 2 Low Tank

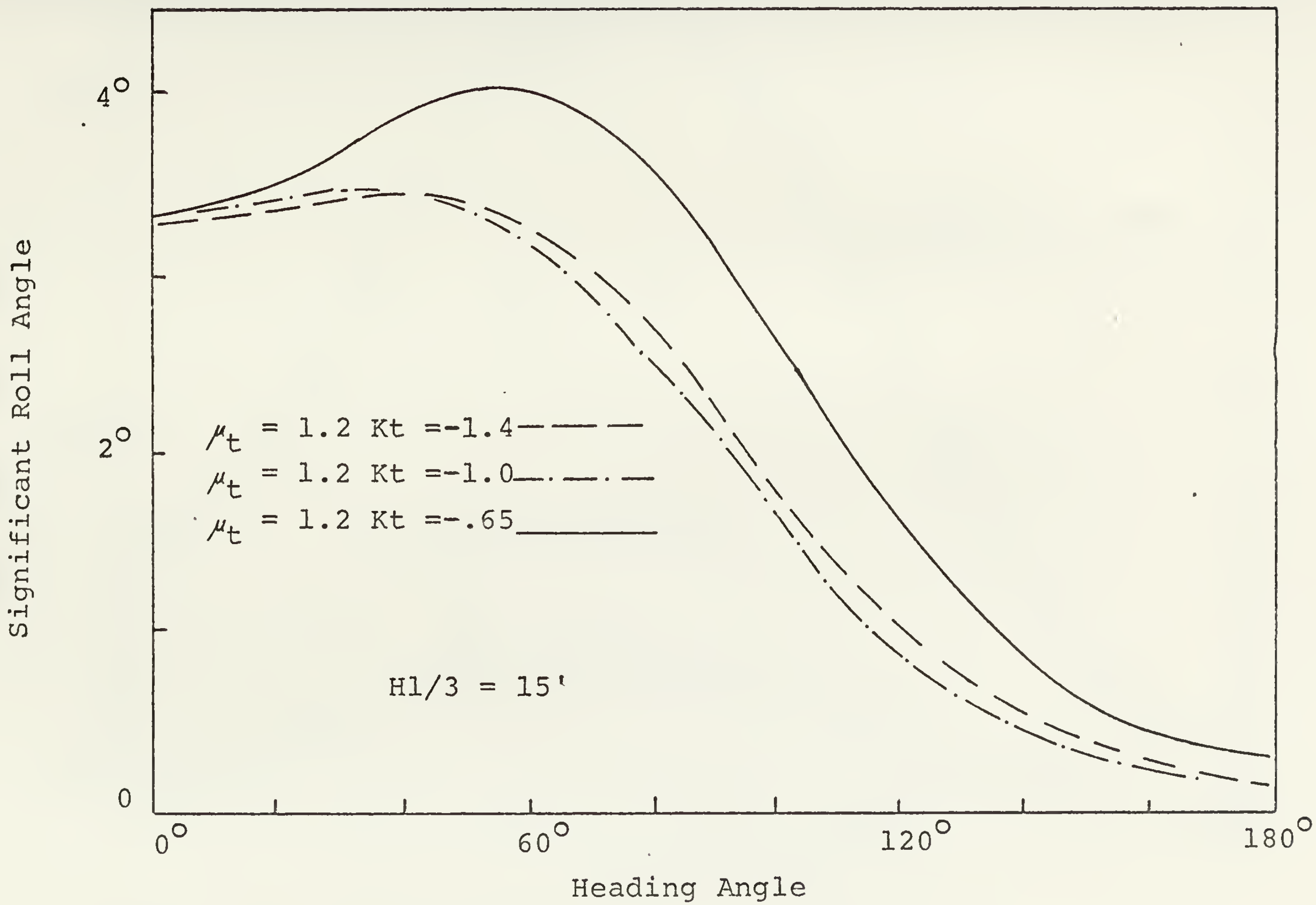


Figure 14C Ship No. 2 High Tank

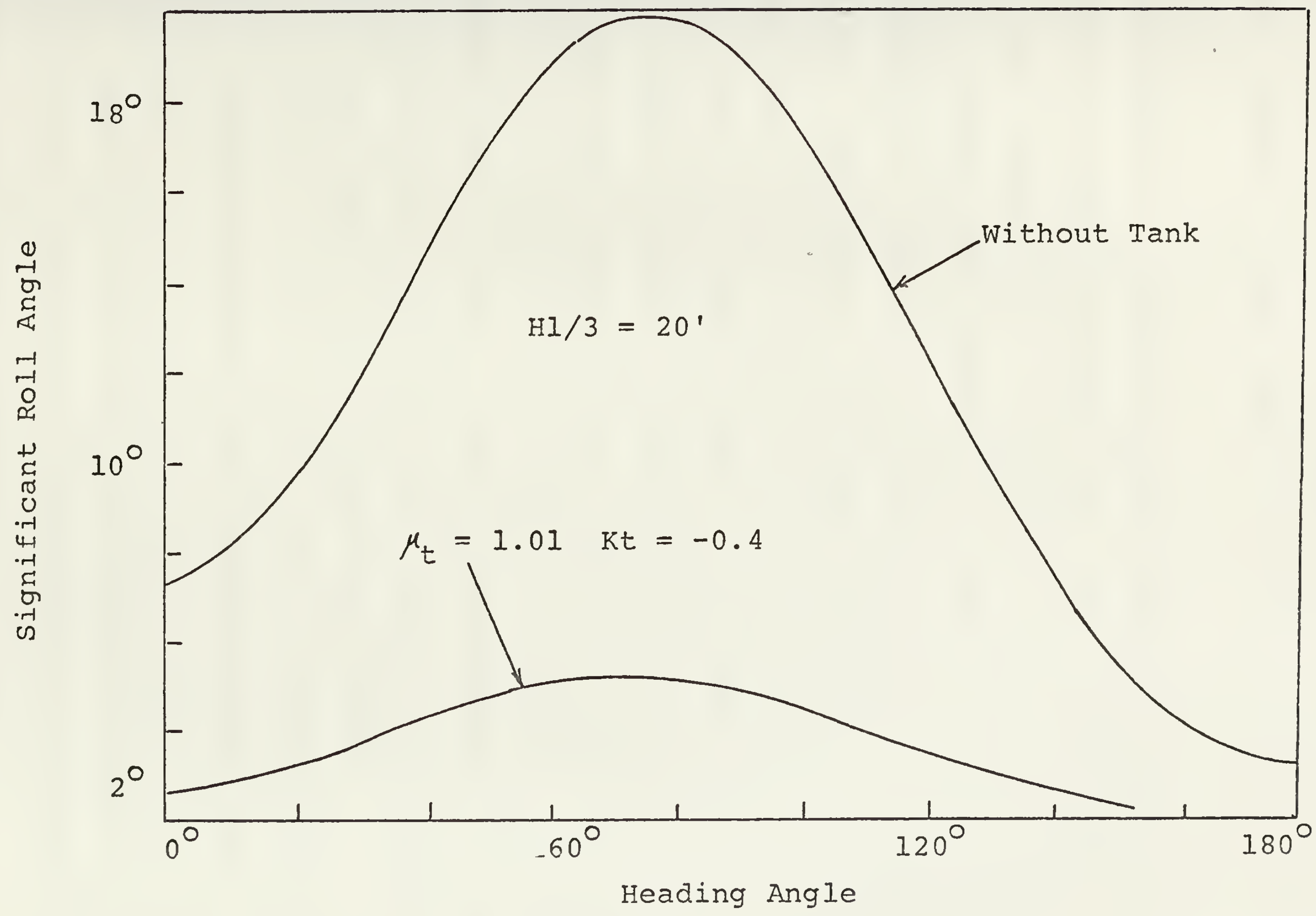


Figure 15 Ship No 3 Low Tank

which was the first iteration guess, was just about optimum. Overall performance of the antirolling tank compared to the ship without a tank was not spectacular, but because the tank uses reserve fuel oil as a working fluid and is partially situated in normal tankage space, it was a reasonably successful design.

On Ship No. 2 the low and high tanks were both carried through to a final iterative step. The initial guess of 118° as a worst heading angle delayed the solution somewhat because at that heading the characteristic of $\phi^{1/3}$ did not flatten out as expected and also 118° was nowhere close to the real worst heading. Both high and low tanks performed well and it is interesting to note that in spite of the much smaller size, weight, and λ_t of the high tank it performed as well as the low tank. This was probably because the parameter $\mu_{st}{}^2$ for the high tank had a negative sign, indicating that roll acceleration pushed the tank fluid in the direction to give a positive tank righting moment. For $\mu_{st}{}^2$ with a positive sign, the case for any tank below the ship center of gravity, the opposite reaction of tank fluid to roll acceleration occurs.

On Ship No. 3 only the low tank was investigated fully, but it can be noticed at the 50° heading angle that the high tank's performance is nearly as good as that of the larger low tank. Again, this was due to the negative sign on $\mu_{st}{}^2$.

A relative comparison of the predicted solutions of the most important tank design parameters is in order.

	Ship No. 1 low tank	Ship No. 2 low tank	Ship No. 2 high tank	Ship No. 3 low tank
Ψ max	5.7^{o}	5.2^{o}	3.2^{o}	5.0^{o}
λ_t	0.20	0.30	0.19	0.25
μ_t	1.01	1.02	1.20	1.01
μ_{st}	2.03	2.72	i5.30	2.06
Weight/100Δ	2.33	3.49	1.57	3.00
K_t	-0.8	-1.0	-1.0	-.4
ε	0.73	0.26	0.25	0.26

A performance parameter ε ($\varepsilon = \phi^{1/3}$ max(tank)/$\phi^{1/3}$ max(no tank)) was added at the bottom of the list, but it is difficult to attach much relative meaning to the parameter because all of the ships had different operational criteria. Even if the criteria were the same, the size differential in the ship's displacements would have made comparisons meaningless except in the case of the tanks on the same ship. The tank capacity, Ψmax, did not enter into the optimization because all of the significant tank angles calculated were less than their saturation angles, the arctan (L_o/R). Of the parameters given above, the first four, Ψmax, μ_t, λ_t, and μ_{st} are the most fundamentally important because they determine the antirolling tank size, dimensions, weight, placement in the ship vertically, and they cannot be changed once the installation is completed. K_t however may be readily adjusted to suit the particular ship and the other two parameters relate to performance or economic matters.

6. Analysis of Results

a. Tank Capacity. The question of how well the tank capacity was utilized may be answered by inspection of the results of significant tank angle at the worst heading in short-crested seas. The predicted tank angle, $\eta^{1/3}$, may be compared to the tank saturation angle, arctan (Lo/R), to give a measure of how much capacity was used at the design conditions. Two tank angle statistics were looked at. $\eta^{1/3}$ is a good engineering estimate of the point at which saturation effects must be taken into account in calculations. $\eta^{1/100}$ was arbitrarily taken as an estimate of when tank saturation effects might just start to be noticed by the crew of the vessel because of the slapping of the tank fluid against the top of the tank. $\eta^{1/3}$ is an output of the MAIN 2 program and $\eta^{1/100}$ is found by the statistics of the random process to be 3.34/2.00 times $\eta^{1/3}$.

	Ship No. 1 low tank	Ship No. 2 low tank	Ship No. 2 high tank	Ship No. 3 low tank
$\eta^{1/3}$	9.81°	7.07°	7.24°	13.48°
$\eta^{1/100}$	16.38°	11.81°	12.09°	22.51°
arctan (Lo/R)	26.47°	16.86°	15.96°	19.24°
$\eta^{1/3}$/arctan(Lo/R)	0.47	0.53	0.58	0.89
$\eta^{1/100}$/arctan(Lo/R)	0.62	0.70	0.76	1.17

An immediate observation is that the low tank on Ship No. 3 is the only one which is making good use of its tank capacity at design conditions. The other designs are all wasting space by building too large a tank for the requirements stated.

b. $\underline{\lambda_t}$. The ship-tank coupling parameter did not by itself seem to influence results much except as it contributed to the tank capacity. Ship size and tank placement had a more definite effect.

c. $\underline{\mu_t}$. The tank tuning parameter had a secondary effect on the results because a higher μ_t corresponding to a higher ω_t is realized by decreasing the area of the cross duct, A1. The lighter cross duct weight allowed for more of the fluid to be in the wing tanks. Consequently, a change in tank tuning to a higher tank frequency made for a tank of slightly greater capacity.

d. $\underline{\mu_{st}}$. The ship-tank decoupling frequency parameter was important for the reasons previously discussed concerning it. A more negative value of $1/\mu_{st}^2$, which is accomplished by moving the tank higher vertically, is always associated with better tank performance for a tank of a given λ_t and capacity Ψmax.

e. Weight Fraction. This parameter can be directly related to the cost of the tank. A complementary parameter, the space fraction, was not given because the data was unavailable for useful ship volume.

f. $\underline{K_t}$. The tank damping parameter falls within the narrow range of about -0.4 to -1.4 for all successful designs. This statement is based on analysis of much more data than is presented in the results section. A convincing reason for the truth of this statement can be found by considering the critical damping for a single degree of freedom system. For a single degree of freedom system critical damping implies that there is no overshoot and no oscillatory response to a step input. Such critical damping

has the K_t value of -2.0. Therefore the ratio of tank damping to critical damping falling in the range of 0.2 to 0.7 is reasonable if the tank is to act as a dynamic vibration absorber. A K_t prediction of greater than -2.0 tells us that the given tank parameters are not consistent with a stabilizing tank. An example of K_t greater than -2.0 is shown on figure 15 for the low tank on Ship no. 1.

g. Tank Placement. There is another aspect of vertical tank placement in addition to comments already given about how it affects λ_t, μ_{st} and GM. A special case of tank operation is whenever the duct is closed for some reason while the tank is full, for example for emergency maintenance. On high tanks especially there is a large GM loss due to the KG rise associated with the large tank fluid weight high in the ship. Figure 16A gives for an example the high tank on Ship No. 2 to show the undesirable effects of this operational condition.

h. Ship Size. It seems obvious from the results to state that a lighter displacement ship such as Ship No. 1 apparently is not as well suited a candidate for an antirolling tank installation as a larger ship. Whether this is due to the center of buoyancy being less submerged or to the inertia of the ship allowing for a higher natural frequency remains to be answered.

7. General Conclusions

Although none of the design examples is complete in itself - primarily because a range of speeds and sea states was not looked at for each ship - it was possible to draw a number of general conclusions from them.

First of all, the equations of motion seem to be valid. Uncoupling sway by setting $\lambda b = 0$ and $1/\mu_{st}^{2} = 0$ gives the classical fourth order differential equation for a coupled ship-tank system. The assumed excitations are the primary departure from the latter, along with the sway coordinate, and hence introduce the greatest possible source of error because the reactive force and moment terms were also taken from classical theory. A lot of numerical accuracy hinges upon the two critical assumptions of uncoupling of roll and sway and application of the excitations at the center of (static) buoyancy. Model testing of some sort is needed to prove or disprove these conjectures.

The general design method seems to be flexible and efficient within the range of interest. This was due to the verification that $\phi^{1/3}$ versus μ_t and K_t, a 'drawable' three dimensional function, is a relatively flat function. This was due to its being an integral function.

The solution of the fifth order differential equations of motion by the design method predicts tank parameter values which are quite similar and in some cases identical to the classical fourth order approach. The roll responses of the two approaches is not significantly different. However, a more refined solution is possible by the methods of this paper. For example, the high tank of Ship No. 2 with $\mu_t = 1.2$ had a lower weight than that for the classically designed tank of $\mu_t = 0.97$ due to the smaller cross duct associated with a higher tank frequency.

VII. Conclusions and Recommendations

1. Conclusions

The design method proposed in this paper seems to offer an attractive, efficient and flexible method for the design of passive antirolling tanks. The analysis for active antirolling tanks led instead to a method for a feasibility study for the use of active tanks. The active tank design method predicts performance of an economical active tank and will select optimum tank parameters for it. However, the design of the pump and the pump controller was left as a separate step. The active tank analysis also demonstrated why active tanks are such unpopular creatures. The small difference in tank angle which the pump is able to achieve and the small amount of active damping which it adds to the equations of motion are not nearly enough to overcome the chief cause of tank efficiency suffering in high sea states -- the saturation of the tank which places an upper limit on the tank righting moment.

The design method for passive antirolling tanks is attractive because it uses a good representation of the real ocean wave process in predicting roll angle. Since there is much available information on ocean conditions for specific shipping routes or operational areas, this information may be substituted for the assumed Pierson-Moskowitz function. The design method is efficient because the equations of motion lend themselves to a complex algebraic form of solution, thereby resulting in a high numerical efficiency for the computer program. Because

of this a large range of solutions may be investigated at little cost. Finally, the design method is flexible in that tank design may be approached from a wide variety of given requirements. For example, meeting some criteria for operational use, meeting space or weight constraints, or any combination of these may be imposed as design requirements. Also, the equations for the hydrodynamic and tank coefficients may be changed as necessary in the main computer program without altering the subroutine which calculates roll response.

The prediction of roll angles seems reasonably well in agreement with all of the references, both in magnitude and in distribution over heading angles. The short-crested sea predictions should be more accurate than those for long-crested or regular waves because linearity is better preserved. Only small components of roll for each heading angle are added together by superposition, including the worst heading at resonance peaks.

A general conclusion concerning design parameters is that a reasonable passive tank design for preliminary design purposes may be estimated for a given λ_t, or $\lambda_t = 0.20$, μ_{st} from Appendix B, μ_t from the Chadwick-Klotter equation on page 53, $K_t = -0.80$, and the tank dimensions from Appendix B.

2. Recommendations

It was decided to first investigate and validate the equations of motion by taking the simplest case of a passive tank and linearized coefficients. As it became apparent that the results of this approach were promising, a design method for passive

tanks was devised. A thorough checking of the results by model or full scale testing seems to be in order as the next step.

The numerical integration technique seems to offer the best hope for modelling a first order, or higher order control effector in conjunction with the fifth order ship and tank equations. However, as previously mentioned this technique is inefficient in that there are so many integration steps to be performed in order to do an optimization, and several orders of magnitude more integration steps to be able to look at short crested seas over a range of sea states and ship speeds. In short, the numerical analysis technique has a certain merit for analysis but is a slumsy synthesis tool. Now that the basic equations for the reactive forces and moments have been validated, there may be a way to overcome this difficulty. This is to combine the spectral density function and the spreading function beforehand to make again an irregular, short-crested sea by taking all the harmonic frequencies and superposing them. A five or ten minute real time segment of excitations could be made and stored as library data. This data could be manipulated for speed, heading angle and sea state to form a simulation of the excitations. This could be used to drive the equations of motion with the effector included. Data for events such as significant roll angle could be obtained from the statistics of the results. It would be interesting to see how well the results of such a simulation compare with the complex algebraic solution. There is room for question as to whether the simulation would be as accurate as the complex algebraic solution because of the reduced

form some of the linear coefficients would assume. For example, the sway damping term must lose its frequency dependence in this type of approach because the excitation would be a series of absolute quantities derived from a frequency spectrum and would lack the frequency information necessary for calculation purposes.

It was noticed when comparing the results of a passive tank design with the design solution contained in reference 13 that there was very close agreement in overall results in short-crested seas except for the case of the region of heading angles near 0° and near 180°; that is, head seas and stern seas. The author believes that the discrepancy most likely could be due to the simplistic approach taken towards obtaining a heading angle correction in Section III.7. This author assumed a constant distribution of wave excitations over an effective shiplength. It is recommended that other distribution functions more closely resembling that which is actually found aboard the ship be tested to see whether a closer agreement could be realized. The implications of using a second or third order parabolic distribution would be worth examining. Before doing this, however, it would be wise to carry through on the first recommendation given in this section and check on which of the predictions of roll angle are closer to model or full scale experimental tests, those of this author or those of reference 13.

A final recommendation is that some of the steps for passive tank design could be better automated. For example, the integrals S' and S" could be accurately handled for any situation

by the proper numerical program. Also, the burden of solving the tank sizing equations on Figure 14 which need an iterative convergent solution should be removed from the tank designer. This would be no simple task of programming, however, because of the great number of combinations of constraints such as ω_t or ω_{st} being limited by tank dimensional constraints.

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Appendix A Anti-Rolling Tank Geometry Factors

It was decided to construct a parametric family of Anti-Rolling tanks using the simple tank geometry of Figure A-1. In practice, a much more complicated geometry could conceiveably exist due to the need to use the ship's bulkheads, sides, and decks as part of the tank structure. Also, depending upon the tank damping required, it may be necessary to fair in sharp corners or other flow obstructions. It is expected, however, that the assumed geometry will be close enough to any required geometry in any specific installation so that equivalent dimensions and equivalent integrals (S' and S") can be quickly calculated and overall dimensions easily estimated.

The basic geometry is shown below.

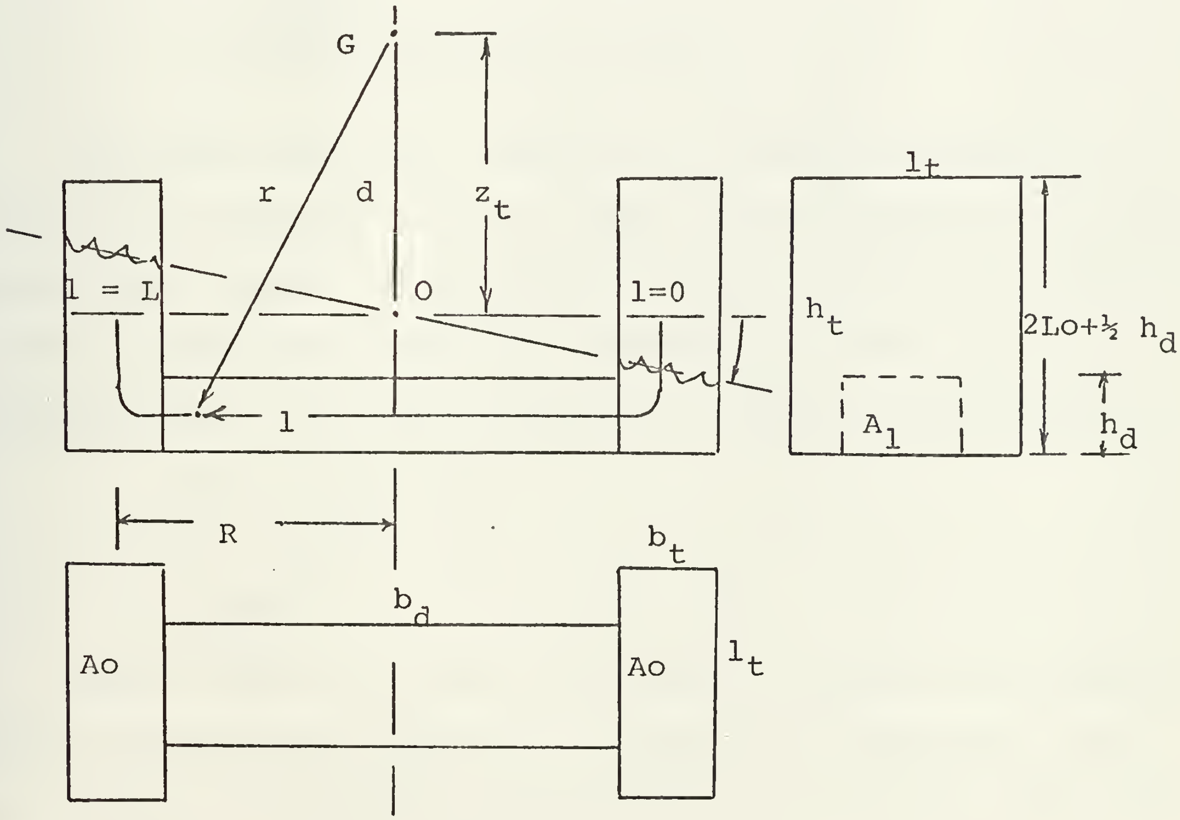


Figure A-1. Anti-Rolling Tank

The tank form factors are S' and S".

$$S' = \int_0^l (Ao/a)\,dl$$

$$S'' = \int_0^l (d/R)\,dl$$

Performing the required integrations by parts over each of the regions of constant sectional area, A_o, A_1, and A_o, and dropping the nearly negligible higher order terms gives the desired results.

$$S' = 2(L_o + (R - b_t)A_o/A_1) \cong 2R(L_o/R + A_o/A_1)$$

$$S'' = 2R(2L_o/R + z_t/R)$$

The two tank natural frequencies are ω_t and ω_{st}.

$$\omega_t = \sqrt{2g/S'}$$

$$\omega_{st} = \sqrt{2g/S''}$$

$$\omega_t \cong \sqrt{g/_R} \cdot \sqrt{/(Lo/R + Ao/A1)}$$

$$\omega_{st} \cong \sqrt{g/_R} \cdot \sqrt{1/(2Lo/_R + 2t/_R)}$$

The above steps have reduced the problem of selecting a tank size to selecting a mix of the tank parameters R, (A_o/A_1), (L_o/R), and (z_t/R). It may be seen that the other tank dimensions are known once these are established. The range of parameters for which this analysis is valid is approximately:

$$0.2 < L_o/R < 1.0$$

$$-1 < z_t/R < +2$$

$$R < B/2$$

Notice that for $z_t < -2L_o$ the value of ω_{st} will be imaginary. Later work with the equations show that this indicates a 90^o phase shift of the motion induced in the tank by the ship roll

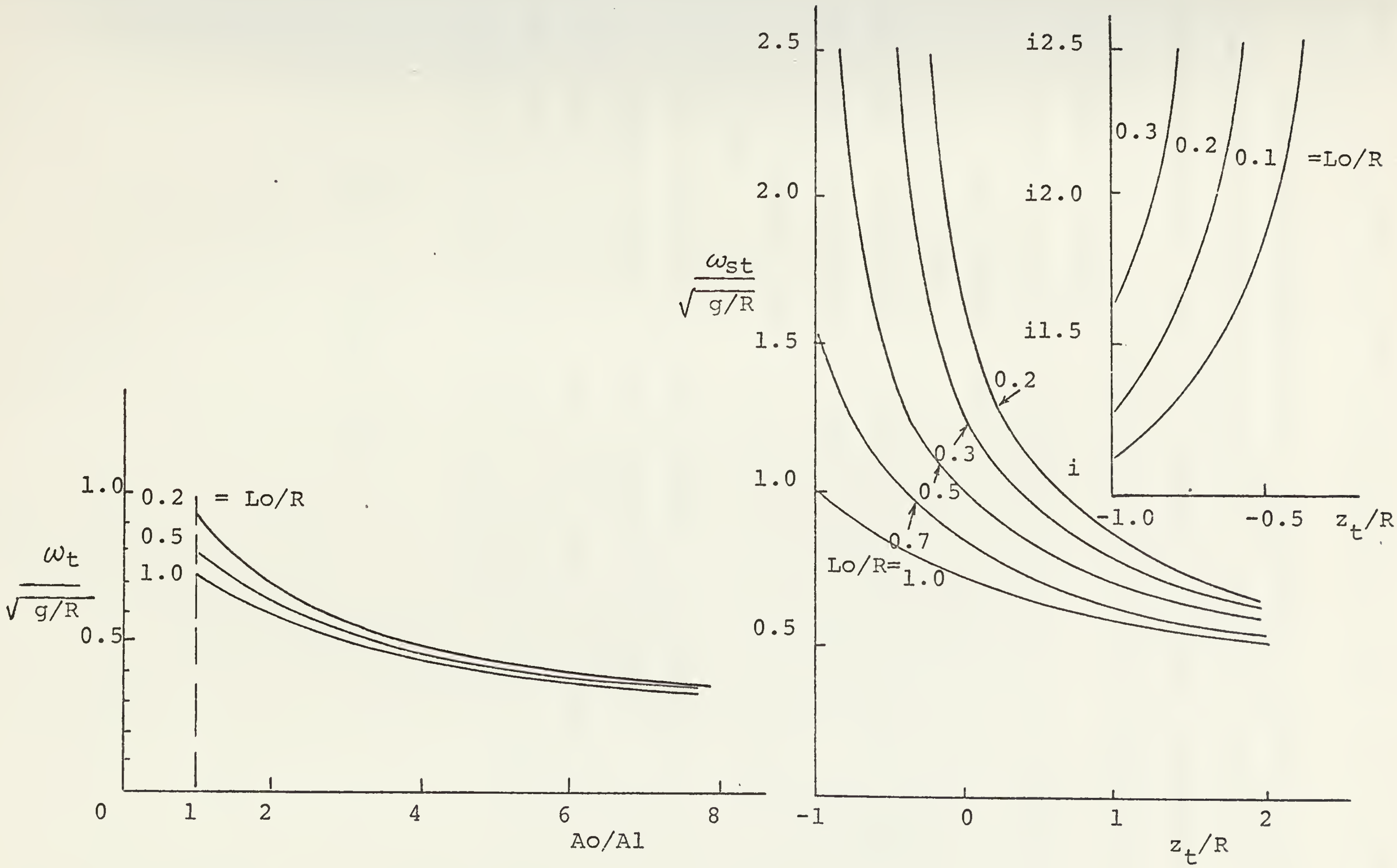


Figure A-2 Tank Natural Frequencies

motion, but does not imply any dramatic change in the equations of motion.

Figure A-2 summarizes the relationship between the tank frequencies and the tank parameters. For most combinations of these parameters, and in the range of validity previously mentioned, Figure A-2 is sufficiently accurate for preliminary tank design purposes.

A convenient way to determine tank size is by fluid weight.

$$\text{Tank weight (fluid)} \cong 2\rho A_o R(L_o/R + A_1/A_o)$$

An effectiveness parameterof tank stabilizing potential is the ratio of tank righting moment to wave upsetting moment per unit of tank or waveslope angle. This ratio, λ_t, is also equal to the percentage reduction in static GM that the tank induces on the ship.

$$\lambda t = 2\rho g A_o R^2/K\phi = \gamma 2 A_o R^2/35\Delta GM \quad (\Delta \text{ in long tons, } \gamma = \rho/\rho H_2 0)$$

Another measure of tank stabilizing potential is the maximum waveslope that the tank could statically stabilize. Since the maximum tank angle possible is about L_o/R (radians), the static tank capacity ψ max is given by:

$$\psi_{max} = \lambda t L_o/R$$

$$\psi_{max} = 2\gamma A_o L_o R/35\Delta GM$$

Appendix B. Coefficient Estimates

1. Hydrodynamic Coefficients

Figures B-1 and B-2 display the hydrodynamic coefficients for ellipsoidal bodies of revolution, as theoretically predicted by potential flow theory. The strategy in using these figures is to assume a body with the same dimensions (L,B,T,) as the ship. In cases where the mass of the body of revolution is quite a bit different than that of the actual ship, correction can be made by taking the average value of the coefficient for a body of the same dimensions (L,B,T) and for a body of the same mass as the ship with similar geometry (L/B, B/T, L/T).

A. $K\dot{p}$ and $Y\dot{v}$

These are both constants. For order of magnitude estimates, $Y\dot{v} = -0.9m$, and $K_{\dot{p}} = -0.25\ I_x$.

B. K_p and Y_v

K_p, given as B44 on the figure, is assumed to be a constant mean value for reasons previously discussed. B_{22} on the figure corresponds to Y_{v1} (ω,U) by the notation of this paper. It is assumed to be well enough represented as a sequence of linear segments. Figure B-3 graphically depicts this linear segmented approximation. The coordinates (F_i, D_i) are used as computer program inputs. The function is assumed constant after point 3; although this is certainly grossly inaccurate, the roll response operator for the normal range of ships is so small in this region that any assumption about the

behavior of the actual function will have essentially no effect on the roll angle solution.

The sway damping component $Y_{v2}(U)$ is estimated by the methods of reference (3) to be:

$$-\pi\rho T^2 U \leq Yv2(U) \leq -(\pi\rho/2)\, T^2 U$$

The lesser absolute value is associated with a bare hull only, and the factor of two is added for a ship with a large amount of rudder surface area and other stabilizing underwater sail areas such as skegs, keels, etc. A particular ship may be somewhere between. For a normal amount of rudder area it will probably be close to the larger absolute value.

C. <u>$K\phi$</u>

$K\phi$ may be found directly from the formula $K\phi = -mg\ GM$.

2. Ship Parameters

A. <u>t_b</u>, the center of buoyancy

The center of buoyancy is accurately estimated by using Morrish's Formula for KB.

$$t_b = T - KB$$

$$KB = 1/3(5/2\ T - 35\Delta/Awp)$$

$$Awp = Cwp \cdot L \cdot B$$

$$Cwp \cong 0.237 + 0.833\ Cp \quad \text{reference 14.}$$

B. <u>GM and G.</u>

The vertical center of gravity and GM, if not known, may be estimated together by the relations:

$$KG = KB + BM - GM$$

$$BM = Cit\ L(B)^3/35\Delta$$

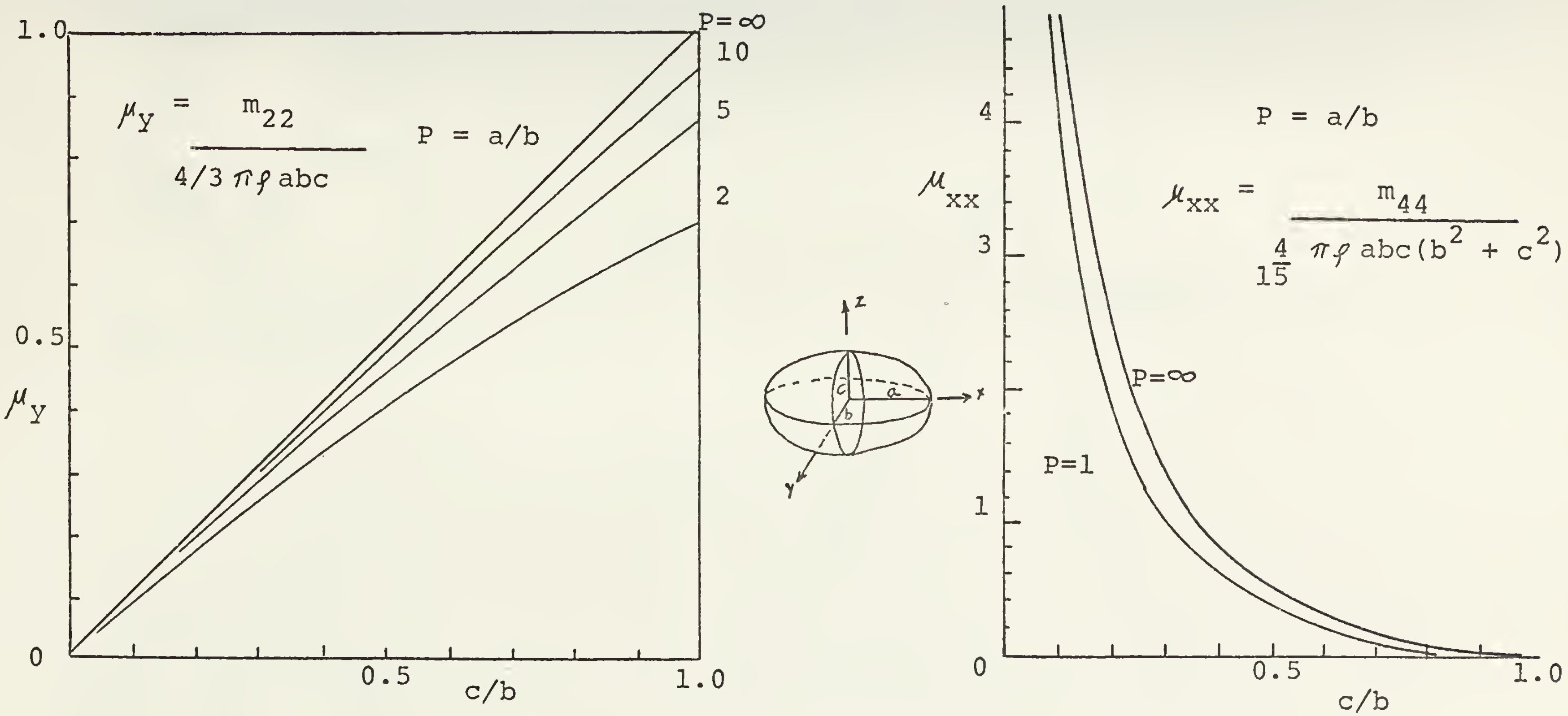
$$Cit \cong -0.013 + 0.10\ Cp$$ reference 14

$GM \cong 0.08\ B$ (see reference 3 for a list of representative GMs)

Notice that if one or the other of G or GM are known, the unknown quantity is very accurately determined. Otherwise, both quantities suffer from being subject to 'guesstimates'.

C. Ix

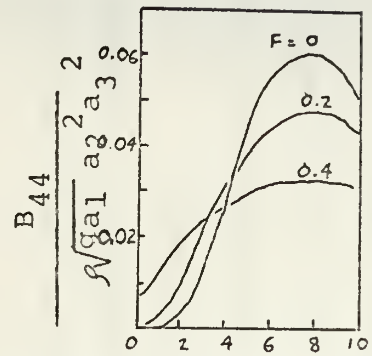
The transverse moment of inertia of the mass of the ship may be closely estimated by: $k_x \cong 0.38\ B$; $Ix = m\ (k_x)^2$



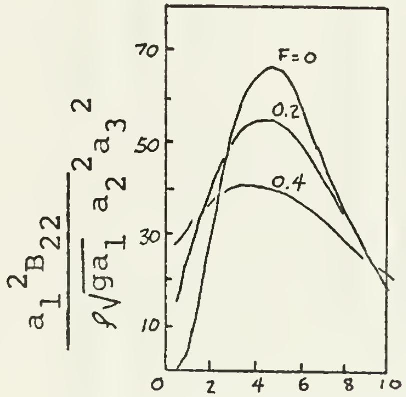
Added mass coefficient of an ellipsoid with three unequal axes, for acceleration along the intermediate axis.

Added moment of inertia coefficient of an ellipsoid, for rotation about the long axis

Figure B-1 $Y\dot{v}$ and $K\dot{p}$ (from reference 13)



Roll damping coefficient for an ellipsoid.



Sway damping coefficient for an ellipsoid, $a_2/a_1 = 1/7$, $a_3/a_1 = 1/14$ $h/a_1 = 2/7$, for various Froude numbers $F = c/(2ga_1)^{\frac{1}{2}}$

Figure B-2 Yv and Kp (reference 13)

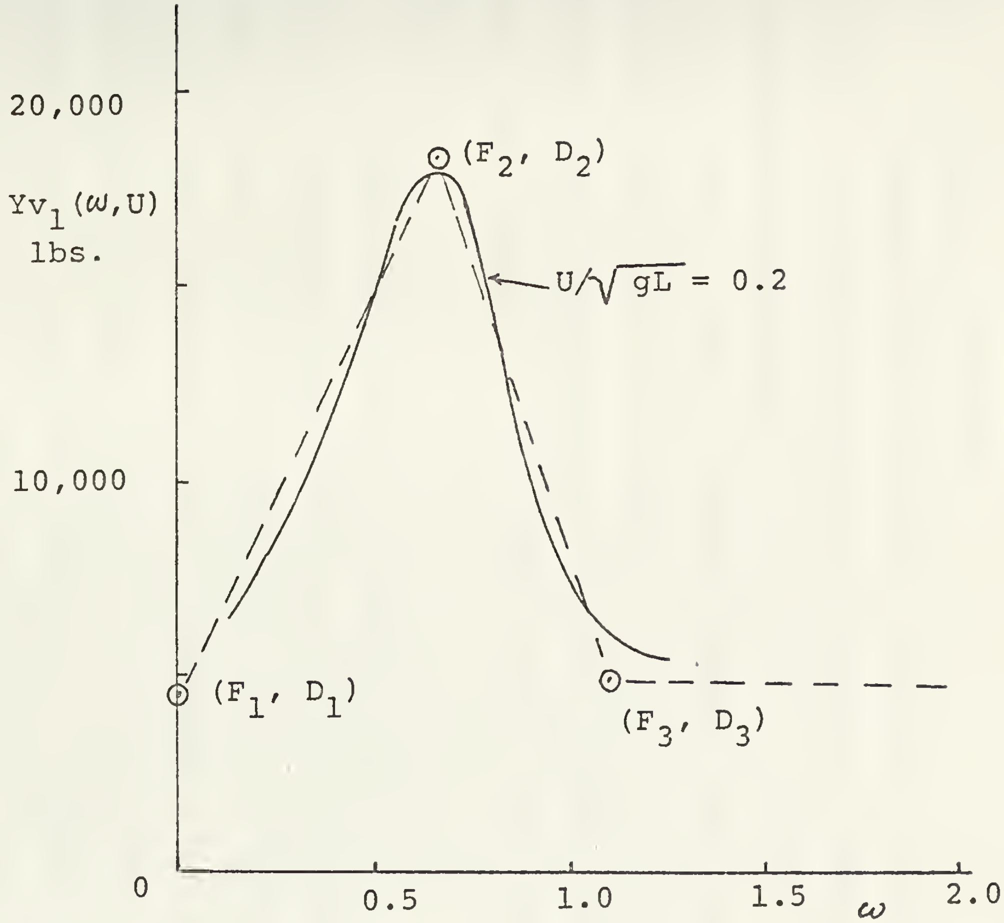


Figure B-3 Yv_1 estimation

Appendix C. Computer Program

1. General Information

A program which provides a numerical solution to the equations of motion was written in standard Fortran language. It is compatible with any Fortran capable system as long as the input device number KI and the output device number KO are correctly specified in the program. A plotting subroutine PICTR was used in the program; if this library subroutine is unavailable, replace the CALL PICTR cards with dummy CONTINUE cards.

There are two basic programs, Main 1 and Main 2, which are separable decks and need not be run concurrently. A single subroutine RESP calculates the roll angle per unit waveslope, the tank angle per unit waveslope, and the phase angle between the tank and roll. A library subroutine, PICTR, is called to give a pictorial display, but because the results are printed out anyway, it is unessential to the program.

Main 1 section of the program computes optimum tank damping and tuning for a given set of conditions.

Main 2 section of the program computes roll response versus heading angle in short crested seas for a given ship, tank, and sea state.

Because there are many untried variations to the program it is advised that anyone using it should proceed with care and check results at each step of the computations.

2. Computer Program Symbols

The following symbol list cross references computer variables with the symbols used elsewhere in this text, where indicated.

MAIN PROGRAM

IX	Ix
LB	λb
KFI	$K\phi$
KPD	$K\dot{p}$
KY	K_y
KS	K_s
LY	λy
KP	K_p
H3	$H^{1/3}$
LE	Le
C	Spreading Function Coefficients
XLAB	Axis Labels
TITLE	Title Tables
XMT	μt
XXMT	Dummy μt
F13	$\phi^{1/3}$
RAO	Response Ampl. Operator
O	ω
S or SK	$\psi\ 2/\delta\omega$
AB	Dummy holder
E	Tank Angle η
TANG	$\eta^{1/3}$
KO	Output device number
KI	Input device number
BG	BG
GM	GM
CB	Cb
XL	L
B	B
T	T
TB	t_b
XYVD	$\|Y\dot{v}/m\|$
XLT	λt
YV^2	Y_v
U	U
XKT1	K_t initial value
DKT1	K_t increment
XMST2	μst^2

MAIN PROGRAM (continued)

DMT	μt increment
W1	ω initial value
DW	ω increment, $\Delta\omega$
XKPD	$\lvert K\dot{p}/Ix \rvert$
BETA	β
D1 - D3	Sway Damping Y_{v1} values
F1 - F3	ω values corresponding to Di
YV	Yv
YVD	$Y_{\dot{v}}$
G1	$\gamma 1$
G2	$\gamma 2$
WS	ω_s
WE	ωe
W	ωo
X1	X
EP	Heading angle correction, 1/2 X
CTB	$e^{-2tb\omega^2/g}$
ETA or E	η
YV1	Y_v
N	Number of increments of $\Delta\omega$

SUBROUTINE RESP

LT	λt
LO	λo
LY	λy
LB	λb
M	μ
M2	μ^2
MT2	μt^2
MST2	μst^2
KY	K_y
KS	K_s
KT	K_t
W	ωo
WS	ωs
G1	$\gamma 1$
G2	$\gamma 2$
TOCBL	Phase angle
Aij	Matrix elements, real
Bij	Matrix elements, imaginary
Ci	Intermediate matrix steps
RX	ϕ/ψ
RT	η/ψ

3. Computer Program Input

The following input data are required by the program.

Ship Dimensions

L	Ship Length
B	Ship Beam
T	Ship Draft
Cb	Block Coefficient (Immersed volume/LBT)
BG	Distance from center of gravity to center of buoyancy, + in z direction
GM	Metacentric Height
t_b	Draft to the center of buoyancy
Le	Effective shiplength (between 60% beam sections)

Tank Parameters

μt	Tank tuning ($\omega t/\omega s$)
$\Delta\mu t$	Tank tuning increment for search routine
K_t	Tank damping
ΔK_t	Tank damping increment for search routine
μst^2	Ship-tank decoupling frequency; primarily a function of vertical placement
λt	Ship-tank coupling parameter -- % loss in static GM of ship due to tank free surface effect

Equation Coefficients

K_p	Roll damping
$\lvert Y_{\dot{v}}/m \rvert$	Added mass coefficient in sway.
$\lvert K_{\dot{p}}/Ix \rvert$	Added inertia coefficient in roll.
D1 - D3	Sway damping ordinates for $Y_{v1}(\omega,U)$
F1 - F3	Sway damping corresponding to D_i

Excitation Parameters

$\omega 1$	Initial value (lower cut-off frequency, about 0.2)
$\Delta\omega$	Increment of ω (notice, 30 increments are used unless N is changed in the program; the 30 increments plus ω_1 will determine the upper cut-off freq.)
$H^{1/3}$	Significant waveheight of seas.
β	Heading angle of ship with respect to the seas (180° for head seas)
U	Ship speed in the x-direction

Program Control Variables

There are three control variables which must be specified on the input deck. They are used to switch various functions in the program, and to terminate the computations.

KSTOP	±1	Read Type 1 data cards (+1, no PICTR) (-1, PICTR at $\mu t = \mu t1$, $Kt = Kt1$ for RAO for MAIN 1) (-1, PICTR of short crested seas for MAIN 2)
	0	Stop execution of the program!!
JSTOP	+1, 0	Main 1 Program executed
	-1	Main 2 Program executed
LSTOP	+1	Read Type 2 data cards
	0	Stop execution of program!!
	-1	Read Type 1 data cards; switch back to Main 1 Program

Examples of sample data decks will resolve any confusion caused by the above statements. The intent of having the switching variables was so as to be able to run Main 1, Main 2, or both together, and at the same time use some of the

subscripted computer variables (matrix elements) as dummy holders for intermediate calculation steps. Unless the data decks examples are followed, there are probably several non-workable combinations of the switching variables.

Data Card Input Deck Format

The general format for all the variables is floating point decimal. the exceptions are JSTOP, KSTOP, and LSTOP, which are fixed point decimal (I2).

There are three different types of data cards. To run Main 1, types 0, 1, & 3 are used. To run Main 2, types 0, 1, 2, & 3 are needed. The data values on types 1 and 2 need not be identical; the reason for having type 2 data cards was to permit a number of variations of data without retyping a whole type 1 deck.

Card #	Column							
0	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80
	(column 1-2 KSTOP)							
	(column 3-4 JSTOP)							
1A	BG	GM	Cb	L	B	T	t_b	$H^{1/3}$
1B	K_p	$\lvert Y_{\dot{v}}/m \rvert$	λt	blank	Le	U	K_{t1}	ΔK_t
1C	μst^2	$\mu t1$	$\Delta_{\mu t}$	$\omega 1$	$\Delta\omega$	$\lvert K_{\dot{p}}/I_x \rvert$	β	blank
1D	D1	D2	D3	F1	F2	F3	blank	blank
1E	/←X-axis label, colums 1-40→/←Y-axis label, columns 41-80→/							
2A	U	$H^{1/3}$	μt	K_t	(column 41-42 LSTOP) 43-80 blank			
2B	/←Title to be printed below short crested sea results, col 1-80→/							
2C	/←X-axis lable colums 1-40→/←Y-axis label, columns 41-80→/							
3	/←blank card→/							

Combinations of data cards for successful computer runs:

1. Find optimum tank parameters (Main 1)

(1A - 1F), (1A - 1F),.............(1A - 1F), 3; set KSTOP = 01, JSTOP = 00

2. Find response versus heading angle (Main 2)

(1A - 1F), (2A - 2C), (2A - 2C).....(1A - 1F), (2A - 2C)....(2A - 2C),3

Set KSTOP = 01 or -1, JSTOP = 01, LSTOP = 01 or -1.

```
C     LINEAR EQUATIONS OF MOTION FOR A PASSIVE ANTI-ROLLING TANK
      REAL IX,LB,KFI,KPD,KY,KS,LY,KP,H3,LE
      DIMENSION C(13),XLAB(20),TITLE(20)
      DIMENSION XMT(3),FI3(3),PAD(60),O(60),S(60),AB(2,60),E(60),TANG(3)
      DATA C(1)/0./,C(2)/.011/,C(3)/.042/,C(4)/.083/,C(5)/.125/
      DATA C(6)/.155/,C(7)/.168/,C(13)/.0/
      DATA C(8)/.155/,C(9)/.125/,C(10)/.083/,C(11)/.042/,C(12)/.011/
      KO=5
      KI=8
      N= 30
   77 READ(KI,916)KSTOP,JSTOP
      IF(KSTOP)79,78,79
   79 READ(KI,907)BG,GM,CB,XL,B,T,TB,H3
      READ(KI,907)KP,XYVD,XLT,YV1,LE,U,XKT1,DKT1
      READ(KI,908)XMST2,XMT(2),DMT,W1,DW,XKPD,BETA
      READ(KI,909)D1,D2,D3,F1,F2,F3
      READ(KI,906)(XLAB(J),J=1,20)
      XM=CB*XL*B*T*1.99
      KFI=-XM*32.2*GM
      IX=XM*(0.4*B)**2
      KPD=-XKPD*IX
      SKFKP=SQRT(KFI*(KPD-IX))
      YV=-3.14*1.99*U*T**2
      YVD=-XYVD*XM
      G1=(B*B)*YVD/(KPD-IX)
      G2=(2.0*YVD-XM)*B*3/(KPD-IX)
      KY=B*B*YV/SQRT(KFI*(KPD-IX))
      KS=KP/SQRT(KFI*(KPD-IX))
      LY=YVD*32.2*B/KFI
      WS=SQRT(KFI/(KPD-IX))
      WST=WS*SQRT(XMST2)
      DETA=BETA*57.3
      LB=BG/B
      IF(JSTOP)81,81,80
   81 WRITE(KO,910)H3,DETA
C     MAIN 1 PROGRAM     INSERT BETWEEN CARDS # 81 & 80
```

```
      WRITE(KO,911)U,LE
      WRITE(KO,912)WS,XLT
      WRITE(KO,913)GM,WST
      WRITE(KO,914)LB,LY,G1,G2
      WRITE(KO,915)KS,KY,XM,IX
      XMT(1)=XMT(2)-DMT
      XMT(3)=XMT(2)+DMT
      FUB=-U*COS(BETA)/32.2
      W=W1
      DO 7 I=1,N
      S(I)=0.00
      EP=1.0
      X1=XLE*ABS(COS(BETA))*W*W/203.2
      IF(0.50-X1)298,299,299
  298 EP=0.25/(X1*X1)
  299 IF(W.LE.0.11)GO TO 296
      IF((0.0324*(32.2/H3)**2/(W**4)).GE.10.0)GO TO 296
      S(I)=EP*(SIN(BETA)**2)*.0081/W*EXP(-.0324*(32.2/H3)**2/(W**4))
      S(I)=S(I)*EXP(-2.*W*W*TB/32.2)
  296 O(I)=W
      WRITE(KO,1000)S(I),W
    7 W=W+DW
      DO4 J=1,3
      WRITE(KO,928)J
      XKT=XKT1
      DKT=DKT1
      JJ=J+1
      IF(JJ.LT.4)GO TO 5
      JJ=1
    5 FIT1=100.
      II=0
    3 W=W1
      FIT=0.00
      ETA=0.00
      DO 2 I=1,N
      WE=ABS(W+W*W*FUB)
```

```
      IF(WE-F2)20,20,21
   20 YV1=(D1+(D2-D1)/F2*WE)
      GO TO 24
   21 IF(WE-F3)22,22,23
   22 YV1=(D2+(D3-D2)/(F3-F2)*(WE-F2))
      GO TO 24
   23 YV1=D3
   24 KY=B*B*(YV1+YV)/SKFKP
      XXMT=XMT(JJ)
      CALL RESP(XLT,XXMT,XKT,W,XMST2,U,FUB,G1,G2,KS,KY,LB,LY,B,WS,TOCBL,
     +RA,RB)
      RAO(I)=RA**2
      E(I)=RB**2
      IF(JJ.NE.2)GO TO 8
      AB(1,I)=W
      AB(2,I)=RA
    8 K=I-1
      IF(I.EQ.1)GO TO 2
      FIT=FIT+0.5*DW*(S(I)*RAO(I)+S(K)*RAO(K))
      ETA=ETA+0.5*DW*(S(I)*E(I)+S(K)*E(K))
    2 W=W+DW
      ZZ=SQRT(FIT)*114.6
      WRITE(KO,901)XKT,ZZ
      IF(KSTOP)76,75,75
   75 IF((FIT1-FIT).GT.0.00004)GO TO 6
      IF((FIT1-FIT).GT.0.000)GO TO 1
      DKT=-0.50*DKT
    6 IF(II.GT.9)GO TO 1
      XKT=XKT+DKT
      II=II+1
      FIT1=FIT
      GO TO 3
    1 FI3(JJ)=2.0*SQRT(FIT)*57.3
      TANG(JJ)=2.0*SQRT(ETA)*57.3
      WRITE(KO,902)XMT(JJ),FI3(JJ),TANG(JJ)
    4 WRITE(KO,903)XKT
```

```
      XMTM=XMT(2)+DMT*((2.5*FI3(1)-4.*FI3(2)+1.5*FI3(3))/(FI3(1)-2.*FI3(
     12)+FI3(3))-2.0)
      FI3M=3.*FI3(1)-3.*FI3(2)+FI3(3)-(2.5*FI3(1)-4.*FI3(2)+1.5*FI3(3))*
     1((XMTM-XMT(2))/DMT+2.)+(0.5*FI3(1)-FI3(2)+.5*FI3(3))*((XMTM-XMT(2)
     2/DMT+2.0)**2)
      IF(XMTM.GT.XMT(3))GO TO 77
      IF(XMTM.LT.XMT(1))GO TO 77
      WRITE(KO,905)XMTM,FI3M
      WRITE(KO,921)KO
      GO TO 77
   76 CALL PICTR(AB,2,XLAB,XSCL,2,N,1,0,4,1,0.0,1)
      GO TO 77
   80 CONTINUE
C     MAIN 2 PROGRAM    INSERT BETWEEN CARDS # 80 & 78
  143 READ(KI,920)U,H3,XXMT,XKT,LSTOP
   82 IF(LSTOP)77,78,83
   83 WRITE(KO,924)U,H3
      WRITE(KO,923)XXMT,XKT,XLT,WS,LB,KS
      WRITE(KO,926)KSTOP
      READ(KI,906)(TITLE(J),J=1,20)
      READ(KI,906)(XLAB(J),J=1,20)
      YV=-3.14*1.99*L*T**2
  141 BETA=0.0001
      DO 150 J=1,13
      O(J)=BETA*57.3
      FUB=-U*COS(BETA)/32.2
      W=W1+0.00001
      DO 100 I=1,N
      X1=XLE*ABS(COS(BETA))*W*W/203.2
      EP=1.
      IF(0.50-X1)198,198,199
  198 EP=0.25/(X1*X1)
  199 SK=0.00
      CTB=0.00
      IF(W.LT.0.11)GO TO 196
      IF((0.0324*(32.2/H3)**2/W**4)-10.)195,195,196
```

```
195 SK=EP*(SIN(BETA)**2)*(.0081/W)*EXP(-.0324*(32.2/H3)**2/(W**4))
    CTB=EXP(-2.0*W*W*TB/32.2)
    SK=SK*CTB
196 S(I)=SK
100 W=W+DW
    FIT=0.00
    ETA=0.00
    W=W1+0.00001
    DO 102 I=1,N
    WE=ABS(W+W*W*FUB)
    IF(WE-F2)120,120,121
120 YV1=(D1+(D2-D1)/F2*WE)
    GO TO 124
121 IF(WE-F3)122,122,123
122 YV1=(D2+(D3-D2)/(F3-F2)*(WE-F2))
    GO TO 124
123 YV1=D3
124 KY=B*B*(YV1+YV)/SKFKP
    CALL RESP(XLT,XXMT,XKT,W,XMST2,U,FUB,G1,G2,KS,KY,LB,LY,B,WS,TOCBL,
   +RA,RB)
    RAO(I)=RA**2
    E(I)=RB**2
    L=I-1
    IF(L.EQ.0)GO TO 102
    FIT=FIT+0.5*DW*(S(I)*RAO(I)+S(L)*RAO(L))
    ETA=ETA+0.5*DW*(S(I)*E(I)+S(L)*E(L))
102 W=W+DW
    IF(FIT-.000001)125,125,126
125 AB(1,J)=0.00
    AB(2,J)=0.00
    GO TO 127
126 AB(1,J)=2.0*SQRT(FIT)*57.3
    AB(2,J)=2.0*SQRT(ETA)*57.3
127 WRITE(KO,925)O(J),AB(1,J),AB(2,J)
150 BETA=BETA+0.2618
    WRITE(KO,921)KSTOP
```

```
      WRITE(KO,923)XXMT,XKT,XLT,WS,LB,KS
      WRITE(KO,926)KSTOP
      DO 153 J=1,13
      KR=J+18
      IF(KR.LE.24)GO TO 154
      KR=KR-24
  154 FIT=0.00
      ETA=C.CC
      DO 155 I=1,13
      K=KR+I-1
      IF(K.LE.24)GO TC 156
      K=K-24
  156 IF(K.LE.13)GO TO 157
      K=K-2*(K-13)
  157 FIT=FIT+C(I)*AB(1,K)
      RAO(J)=FIT
  155 ETA=ETA+C(I)*AB(2,K)
  153 WRITE(KO,925)O(J),FIT,ETA
      WRITE(KO,927)(TITLE(J),J=1,20)
      IF(KSTCP.GE.0)GC TO 143
      DO 158 J=1,13
      AB(2,J)=RAO(J)
  158 AB(1,J)=O(J)
      CALL PICTR(AB,2,XLAB,XSCL,2,13,1,0,4,1,0.0,1)
      GO TO 143
   78 CONTINUE
  901 FORMAT(20X,F8.3,F8.3)
  902 FORMAT(22X,'MT= ',F6.2,' FI1/3= ',F6.2,' ETA1/3= ',F6.2)
  903 FORMAT(22X,' TANK LINEAR DAMPING = ',F6.2)
  904 FORMAT(22X,'SI2/DW= ',F9.6,' W= ',F8.5)
  905 FORMAT(//,22X,'MT(MIN)= ',F5.2,' FI1/3= ',F7.4)
  906 FORMAT(20A4)
  907 FORMAT(8F10.5)
  908 FORMAT(7F10.5)
  909 FORMAT(6F10.5)
  910 FORMAT('1',24X,'SIG WAVE HT= ',F5.1,' HEADING ANGLE= ',F7.1,' DEG
```

```
    +')
 911 FORMAT(22X,'SHIP SPEED= ',F5.2,' EFFECTIVE SHIP LENGTH= ',F9.1)
 912 FORMAT(22X,'SHIP NAT FREQUENCY=',F7.4,' SHIP-TANK COUPLING= ',F7.4
    +)
 913 FORMAT(22X,'GM= ',F7.3,' SHIP-TANK DECOUPLING FREQ= ',F5.2)
 914 FORMAT(22X,'LB= ',F7.4,' LY= ',F7.4,' G1= ',F7.4,' G2= ',F7.4)
 915 FORMAT(22X,'KS= ',F7.4,' KY= ',F7.4,' MASS= ',F9.0,' IX= ',F12.0)
 916 FORMAT(2I2)
 920 FORMAT(4F10.6,I2)
 921 FORMAT('1',25X,'SHORT-CRESTED SEAS',10X,I5)
 923 FORMAT('0',27X,'MT= ',6X,'KT=',7X,'LT=',/,28X,F5.2,5X,F5.2,5X,F5.2
    +,/,28X,'WS=        LB=        KS=',/,28X,F5.2,5X,F5.2,5X,F5.2)
 924 FORMAT('1',25X,'LONG-CRESTED SEAS: SPEED= ',F5.2,' SIG WAVE HT= ',
    +F5.1)
 925 FORMAT('0',25X,F5.1,12X,F6.2,12X,F6.2)
 926 FORMAT('0',22X,'HEADING ANGLE   SIG ROLL ANGLE   SIG TANK ANGLE (D
    +EGREES)',I15)
 927 FORMAT('0',22X,20A4)
 928 FORMAT(//,22X,'  KT     FI1/3',15X,I2)
1000 FORMAT(22X,'SI2/DW= ',F9.6,' W= ',F8.5)
     STOP
     END
```

```
 SUBROUTINE RESP(LT,MT,KT,W,MST2,U,FUB,G1,G2,KS,KY,LB,LY,B,WS,TOCBL
+,RX,RT)
 REAL LO,LY,M2,MT2,KY1
 REAL LT,MT,KT,M,KE,KS,KY,MST2,MYT2,LB
 MYT2=32.2/B
 MT2=MT**2
 M=ABS(W+W*W*FUB)/WS
 M2=M*M
 A11=1.-M2*(1.0+LB*LB*G1)
 B11=(KS+(LB**2)*KY)*M
 A12=-2.0*LB*KY
 B12=-2.0*LB*G1*M
 A13=LT*(1.-M2/MST2)
 A21=2.0*LB*G1*M2
 B21=-2.0*LB*KY*M
 A22=2.0*KY
 B22=M*G2
 A23=LT*M2/MYT2
 B32=-LT*M/MYT2
 A33=LT*(1.-M2/MT2)
 B33=LT*KT*M/MT
 C1=A22*A33-B22*B33
 C2=B12*B33-A12*A33
 C3=A22*B33+A33*B22-B32*A23
 C4=A13*B32-A33*B12-A12*B33
 LO=KY*(32.2/(WS*B*(W+0.00001)))
 DR=A11*(A22*A33-B22*B33)-B11*(B22*A33+A22*B33)+A13*(A12*A23-B21*B3
12)-A22*(A13**2)+A23*B32*B11-A12*(A21*A33-B21*B33)+B12*(B21*A33+A21
2*B33)
 DI=A11*(A22*B33+B22*A33)-B11*(B22*B33-A22*A33)+A13*(B12*A23+A21*B3
12)-B22*(A13**2)-A23*B32*A11-A12*(A21*B33+B21*A33)+B12*(B21*B33-A21
2*A33)
 XR=C1+LY*(C2-LB*C1)+LO*(LB*C3-C4)
 XI=C3+LY*(C4-LB*C3)+LO*(C2-LB*C1)
 RX=SQRT((XR*DR+XI*DI)**2+(XI*DR-XR*DI)**2)/(DR**2+DI**2)
 ER=LY*(A12*A13+B11*B32)+LO*(A11*B32-A13*B12)+(LB*LY-1.)*(B21*B32+A
```

```
+22*A13)+LB*LO*(A21*B32-A13*B22)
 EI=LY*(A13*B12-A11*B32)+LO*(A12*A13-B11*B32)+(LB*LY-1.)*(A13*B22-A
+21*B32)+LB*LO*(A13*A22+B21*B32)
 RT=SQRT((ER*DR+EI*DI)**2+(EI*DR-ER*DI)**2)/(DR**2+DI**2)
C     TOCBL=ATAN((XI*DR-XR*DI)/(XR*DR+XI*DI))*57.3    FOR PHASE ANGLE
 TOCBL=1.0
 RETURN
 END
```

```
C     SAMPLE DATA DECK
C
C
C
C
C
C100
9.75       3.36       .4377      408.       45.        14.5       5.67       10.
-.221E07   .67        .2                    269.       43.        -.4        -1.
4.11       1.01       .5         .25        .1         .263       1.571
-8257.     -21450.    -10300.    0.         .684       1.05
  WAVE FREQUENCY  RAD/SEC                      ROLL ANGLE PER UNIT WAVESLOPE
-101
16.5       7.39       .5034       585.      76.67       21.33      8.24      15.
-2.92E07   .53        .001                  439.       34.        -.4        -1.
4.72       1.2        .275       .2         .077       .412       1.047
-30200.    -78500.    -37700.    0.         .684       1.05
  WAVE FREQUENCY  RAD/SEC                      ROLL ANGLE PER UNIT WAVESLOPE
34.        15.        1.2        -25.       01
   SHIP # 2    WITHOUT TANK
STERN SEAS    HEADING ANGLE    HEAD SEAS   SIGNIFICANT ROLL ANGLE  (DEGREES)
                                            -1
43.        10.        1.01       -1.        01
   SHIP # 1    LOW TANK   LT = .20    KT = -1.0
STERN SEAS    HEADING ANGLE    HEAD SEAS   SIGNIFICANT ROLL ANGLE  (DEGREES)
C     BLANK CARD
```

7. Sample Output Data

```
     SIG WAVE HT =  10.0 HEADING ANGLE = 90.0 DEG
SHIP SPEED = 43.00 EFFECTIVE SHIP LENGTH = 260.0
SHIP NAT FREQUENCY = 0.5142 SHIP-TANK COUPLING = 0.2000
GM=  3.360  SHIP-TANK DECOUPLING FREQ=  1.04
LB = 0.2167 LY= 8.9732 G1=  3.3155  G2= 11.5796
KS = 0.0453 KY=-2.33446  MAS=   231884. IX=   75130336.

  KT       FI1/3                 1
-0.400     3.544
-1.400     5.783
-0.900     4.870
-0.400     3.544
 0.100     4.966
-0.150     4.401
-0.400     3.544
-0.650     4.089
-0.525     3.733
-0.400     3.544
-0.275     3.706
MT=   1.01   F11/3=    3.71  ETA1/3=    5.67
 TANK LINEAR  DAMPING = -0.27

  KT       FI1/3                 2
-0.400     3.474
-1.400     4.202
-0.900     3.602
-0.400     3.474
 0.100     5.162
-0.150     4.300
-0.400     3.474
-0.650     3.374
-0.900     3.602
-0.775     3.472
-0.650     3.374
MT=   1.51   FI1/3=    3.37 ETA1/3=      6.33
 TANK LINEAR DAMPING = =0.65

  KT       FI1/3                 3
-0.400     7.451
-1.400     6.939
-2.400     6.430
-3.400     6.190
-4.400     6.067
-5.400     5.996
-6.400     5.952
-7.400     5.922
MT=   0.51   fI1/3-    5.92 ETA1/3=      0.32
 TANK LINEAR DAMPING =  -7.40

MI(MIN)+   1.35 FI1/3= -1.8098
```

LONG CRESTED SEAS; SPEED= 34.00 SIG WAVE HT= 15.0)

MT=	KT=	LT=
1.20	-1.00	0.20
WS=	LB=	KS=
0.40	0.22	-0.06

HEADING ANGLE	SIG ROLL ANGLE	SIG TANK ANGLE (DEGREES)	-1
0.0	0.00	0.00	
15.0	2.23	4.27	
30.0	4.61	9.34	
45.0	6.92	15.78	
60.0	4.44	13.33	
75.0	2.08	8.28	
90.0	1.29	5.59	
105.0	0.87	3.99	
120.0	0.59	2.83	
135.0	0.39	1.93	
150.0	0.24	1.20	
165.0	0.11	0.58	
180.0	0.00	0.00	

SHORT CRESTED SEAS -1

MT=	KT=	LT=
1.20	-1.00	0.20
WS=	LB=	KS=
0.40	0.22	-0.06

HEADING ANGLE	SIG ROLL ANGLE	SIG TANK ANGLE (DEGREES)	-1
0.0	3.41	7.58	
15.0	3.42	7.77	
30.0	3.44	8.23	
45.0	3.40	8.68	
60.0	3.19	8.71	
75.0	2.75	8.08	
90.0	2.14	6.83	
105.0	1.48	5.25	
120.0	0.93	3.72	
135.0	0.57	2.52	
150.0	0.37	1.72	
165.0	0.26	1.27	
180.0	0.23	1.13	

SHIP # 2 HIGH TANK LT=0.20

Appendix D. Other Solution Methods

1. Analog Solution

The equations of motion were put into their proper non-dimensional form for electrical analog simulation. However, it was considered necessary to further investigate the numerical solution in order to properly normalize the non-linear variables, and especially coefficient expressions. Using the results of this paper, an analog simulation is a trivial problem if the modeling of the coefficients can be accomplished.

2. Numerical Integration

In order to perform this technique, the three coupled equations forming a fifth-order system must be algebraically broken down into five coupled first order differential equations. This was done using the set of dimensionless equations in preference to the dimensional set. The five state variables chosen to represent the fifth-order system were: ϕ, $\dot{\phi}$, v^*, η, and $\dot{\eta}$. One of the excitation terms, the $i\lambda_o$ term, was dropped to make the algebra a little more tractable. A parametric study was done in order to justify this step. The below figure shows that for the test case investigated for a "normal" sized ship, the $i\lambda_o$ term has practically no bearing on the excitation for $\omega_o \geq 0.2$, which was previously shown to be the lower boundary of the excitation frequency

range of interest. Therefore, the sway excitation term was shortened to $\lambda_y\psi$ and roll excitation to $(1-\lambda_b\lambda_y)\psi$. This kept the excitation term uni-dimensional.

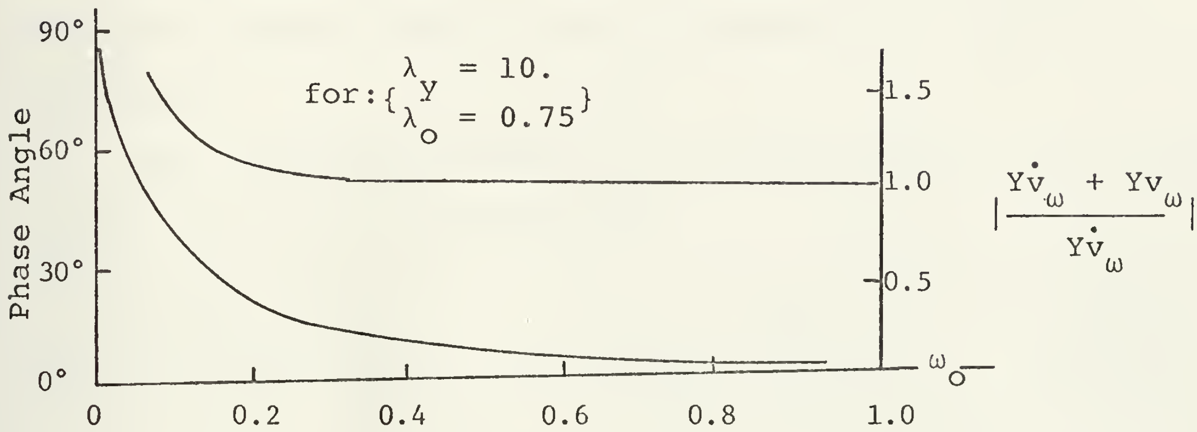


Figure D1. Sway Excitations.

The five coupled ordinary differential equations, in matrix form, were found to be,

$$d/d\tau \begin{bmatrix} \dot{\phi} \\ \phi \\ v^* \\ \dot{\eta} \\ \eta \end{bmatrix} \begin{matrix} = \\ = \\ = \\ = \\ = \end{matrix} \begin{bmatrix} \beta_1/\beta_o & \beta_2/\beta_o & \beta_3/\beta_o & \beta_4/\beta_o & \beta_5/\beta_o \\ 1 & 0 & 0 & 0 & 0 \\ z_1/\gamma_2 & z_2/\gamma_2 & z_3/\gamma_2 & z_4/\gamma_2 & z_5/\gamma_2 \\ \delta_1/\alpha_2 & \delta_2/\alpha_2 & \delta_3/\alpha_2 & \delta_4/\alpha_2 & \delta_5/\alpha_2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi} \\ \phi \\ v^* \\ \dot{\eta} \\ \eta \end{bmatrix} +$$

$$\begin{bmatrix} \beta_6/\beta_o \\ 0 \\ z_6/\gamma_2 \\ \delta_6/\alpha_2 \\ 0 \end{bmatrix} \psi(\tau) + \begin{bmatrix} \beta_7/\beta_o \\ 0 \\ z_7/\delta_2 \\ \delta_7/\alpha_2 \\ 0 \end{bmatrix} p^* + \begin{bmatrix} -K_Q/\beta_o & -1/\phi_R^{\,2}\beta_o \\ 0 & 0 \\ z_8/\gamma_2 & z_9/\gamma_2 \\ \delta_8/\alpha_2 & \delta_9/\alpha_2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} p|p| \\ \phi^3 \end{bmatrix}$$

The above matrix equation is separated into a ship-tank linear matrix, and excitation matrix, a controller matrix, and a ship-tank non-linear matrix. The coefficients in the matrices are all constants for a given set of conditions. A separate equation for p* should be added for a particular contoller.

The identities for the 29 new coefficients, β_o-β_7, z_1-z_9, δ_1-δ_9, and α_1-α_3 are given below.

$$\alpha_1 = [\frac{\lambda_t \lambda_b \gamma_1}{\mu_{yt}^2 \gamma_2} - \frac{\lambda_t}{\mu_{st}^2}] \qquad \alpha_2 = [\frac{\lambda_t}{\mu_t^2} - \frac{\lambda_t^2}{\mu_{yt}^4 \gamma_2}] \qquad \alpha_3 = \frac{\alpha_1}{\alpha_2}$$

$$\beta_o = [1 - \alpha_3 \alpha_1 + \lambda_b^2 \gamma_1 - \frac{\lambda_b^2 \gamma_1^2}{\gamma_2}]$$

$$\beta_1 = [K_t (\frac{\lambda_b^2 \gamma_1}{\gamma_2} + \alpha_3 \frac{\lambda_t \lambda_b K_y}{\mu_{yt}^2 \gamma_2}) - K_s] \qquad \beta_2 = -[1 + \alpha_3 \lambda_t]$$

$$\beta_3 = K_y [\lambda_b (1 - \frac{\gamma 1}{\gamma_2}) - \frac{\alpha_3 \lambda_t}{\mu_{yt}^2 \gamma_2}] \qquad \beta_4 = \frac{-\alpha_3 \lambda_t K_t}{\mu_t} \qquad \beta_5 = -\lambda_t (1 + \alpha_3)$$

$$\beta_6 = [1 + \lambda_b \lambda_y + \frac{\lambda_y \alpha_3 \lambda_t}{\mu_{yt}^2 \gamma_2} + \lambda_b \frac{\gamma_1}{\gamma_2}] \qquad \beta_7 = \alpha_3 \lambda_t$$

$$\delta_1 = [\alpha_1 \frac{\beta_4}{\beta_o} - \frac{\lambda_t K_t}{\mu_t}] \qquad \delta_5 = [\alpha_1 \frac{\beta_5}{\beta_o} - \lambda_t]$$

$$\delta_6 = \frac{\alpha_1}{\beta_o}[1 + \lambda_b \lambda_y + \lambda_y (\frac{\alpha_3 \lambda_t}{\mu_{yt}^2 \gamma_2} + \frac{\lambda_b \gamma_1}{\gamma_2})] + \frac{\lambda_t \lambda_y}{\mu_{yt}^2 \gamma_2}$$

$$\delta_7 = [\alpha_1 \frac{\beta_7}{\beta_o} + \lambda_t] \qquad \delta_8 = \frac{-\alpha_1 k_Q}{\beta_o} \qquad \delta_9 = \frac{\alpha_1}{\phi_R^2 \beta_o}$$

$$z_1 = [\frac{\lambda_b \gamma_1 \beta_1}{\beta_o} + \lambda_b K_y + \frac{\lambda_t \delta_1}{\mu_{yt}^2 \alpha_2}] \qquad z_2 = [\frac{\lambda_b \gamma_1 \beta_2}{\beta_o} + \frac{\lambda_t \delta_2}{\mu_{yt}^2 \alpha_2}]$$

$$z_3 = [\frac{\lambda_b \gamma_1 \beta_3}{\beta_o} - K_y + \frac{\lambda_t \delta_3}{\mu_{yt}^2 \alpha_2}] \qquad z_4 = [\frac{\lambda_b \gamma_1 \beta_4}{\beta_o} + \frac{\lambda_t \delta_4}{\mu_{yt}^2 \alpha_2}]$$

$$z_5 = [\frac{\lambda_b \gamma_1 \beta_5}{\beta_o} + \frac{\lambda_t \delta_5}{\mu_{yt}^2 \alpha_2}] \qquad z_6 = [\frac{\lambda_b \gamma_1 \beta_6}{\beta_o} + \lambda_y + \frac{\lambda_t \delta_6}{\mu_{yt}^2 \alpha_2}]$$

$$z_7 = [\frac{\lambda_b \gamma_1 \beta_7}{\beta_o} + \frac{\lambda_t \delta_7}{\mu_{yt}^2 \alpha_2}] \qquad z_8 = [\frac{-\lambda_b K_Q \gamma_1}{\beta_o} + \frac{\lambda_t \delta_8}{\mu_{yt}^2 \alpha_2}]$$

$$z_9 = [\frac{\lambda_b \gamma_1 (1/\phi_R^2)}{\beta_o} + \frac{\lambda_t \delta_9}{\mu_{yt}^2 \alpha_2}]$$

The matrix equations may be drawn as a signal flow graph in block form. The signal flow graph is useful for interpreting exactly what the basic relationships are between all the variables. This information cannot be extracted from the simpler form of the three coupled equations.

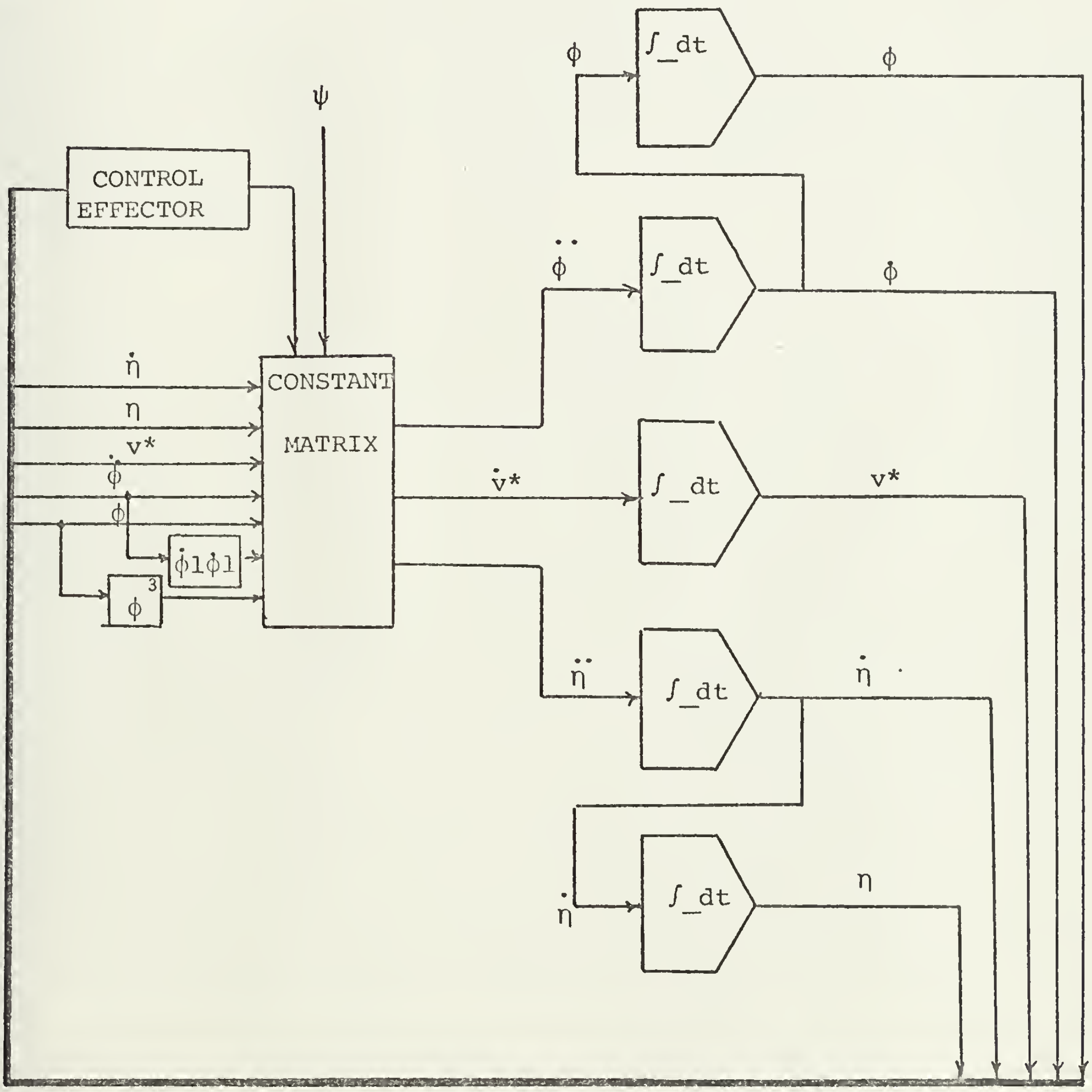


Figure D2. Ship Anti-Rolling Tank System

Simulation of roll by the use of the above equations without the necessity of directly using the spectral density function technique could be accomplished by taking an excitation made of randomly phased harmonic waves, in their proper proportions, and integrating for a ship time duration of enough cycles, say a thousand, to get a meaningful sample for determining ϕ_{rms}. In the interests of amount of effort required, this should be done subsequently to finding the approximate solution by the complex algebraic method adopted by the author.

Thesis
Z365 Ziegler 160792
Anti-rolling tank design.